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Radio mean labeling of Path and Cycle related graphs

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Abstract

A Radio Mean labeling of a connected graph G is a one to one map h from the vertex set V(G) to the set of natural numbers N such that for any two distinct vertices x and y of G, $d(x, y) + \left\lceil \frac{h(x)+h(y)}{2} \right\rceil \geq 1 + diam(G)$. The radio mean number of h, rmn(h), is the maximum number assigned to any vertex of G. The radio mean number of G, rmn(G), is the minimum value of rmn(h) taken over all radio mean labelings h of G. In this paper we find the radio mean number of triangular ladder graph, $P_n \odot \overline{K}_2$, $K_n \odot \overline{K}_2$ and $W_n \odot \overline{K}_2$.

Keywords: Radio mean labeling, Distance, Eccentricity, Diameter, triangular ladder graph.

1. Introduction and definitions

Throughout this paper we consider finite, simple, undirected and connected graphs. Let V(G) and E(G) respectively denote the vertex set and edge set of G. Radio labeling, or multilevel distance labeling, is motivated by the channel assignment problem for radio transmitters [1]. Ponraj et al. [3] introduced the notion of radio mean labeling of graphs and investigated radio mean number of some graphs [11]. D.S.T. Ramesh, A. Subramanian and K. Sunitha investigated radio number for some graphs [9, 10] and introduced the radio mean square labeling of some graphs [8]. The span of a labeling h is the maximum integer that h maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. For standard terminology and notations we follow Harary [4] and Gallian [7]. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter of G.

Definition 1.1[2] The distance d(u, v) from a vertex u to a vertex v in a connected graph G is the minimum of the lengths of the u-v paths in G.

Definition 1.2[2] The eccentricity e(v) of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.

Definition 1.3[2] The diameter diam(G) of G is the greatest eccentricity among the vertices of G.

Definition 1.4 [5] A triangular ladder TL_n, $n \geq 2$ is a graph obtained from a ladder L_n by adding the edges $y_i x_{i+1}$ for $1 \leq i \leq n-1$, where x_i and y_i, $1 \leq i \leq n$, are the vertices of L_n such that $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$ are two paths of length n in L_n.

2. Main Results for path related graphs

Theorem 2.1 $rmn(TL_n) = 4n-3$, $n \geq 2$.

Proof: Let $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$ be two paths of length n. Join x_i and y_i, $1 \leq i \leq n$, the resultant graph is L_n. Join y_i and x_{i+1}, $1 \leq i \leq n-1$. The resultant graph is TL_n whose edge set is $E = \{x_i x_{i+1}, y_i y_{i+1}, y_i x_{i+1} / 1 \leq i \leq n-1\} \cup \{x_i y_i / 1 \leq i \leq n\}$ and diam $(TL_n) = n$.

Define a function $h: V(TL_n) \to N$ by

$h(x_1) = 1$; $h(x_i) = 2n + i - 3$, $2 \leq i \leq n$;

$h(y_i) = 3n + i - 3$, $1 \leq i \leq n$

Now we check the radio mean condition for h.

Case a: Consider the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n + i + j - 6}{2} \right\rceil \geq n + 1 = 1 + diam(TL_n)$$

Case b: Consider the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i) + h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n + i + j - 6}{2} \right\rceil \geq n + 1$$

Case c: Consider the pair (x_i, y_j), $1 \leq i, j \leq n$

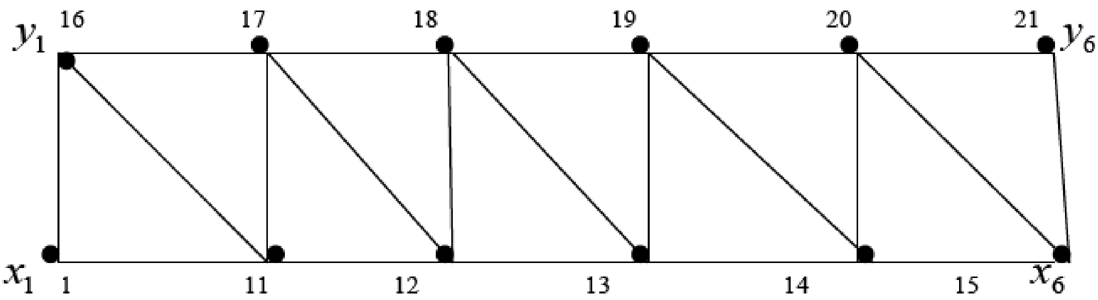
$$d(x_i, y_j) + \left\lceil \frac{h(x_i) + h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{5n + i + j - 6}{2} \right\rceil \geq n + 1$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of TL_n. Therefore $rmn(TL_n) \leq rmn(h) = 4n - 3$

Since h is injective, $rmn(TL_n) \geq 4n - 3$ for all radio mean labelings h and hence

$rmn(TL_n) = 4n - 3$, $n \geq 2$.

Example 2.1



$rmn(TL_6) = 21$

Figure 1

Theorem 2.2 $rmn(P_n \odot \overline{K_2}) = 4n-3$, $n \geq 3$.

Proof. Let $x_1, x_2, \ldots, x_n$ be the path P_n and let y_i, z_i be the vertices of $\overline{K_2}$ which are joined to the vertex x_i of path P_n, $1 \leq i \leq n$. The resultant graph is $P_n \odot \overline{K_2}$) whose edge set is

$E = \{x_i x_{i+1} / 1 \leq i \leq n - 1\} \cup \{x_i y_i, x_i z_i / 1 \leq i \leq n\}$ and $diam(P_n \odot \overline{K_2}) = n + 1$

Define a function $h: V(P_n \odot \overline{K_2})) \rightarrow N$ by

$h(x_i) = 3n+i-3$, $1 \leq i \leq n$;

$h(y_i) = n+i-3$, $1 \leq i \leq n$;

$h(z_i) = 2n+i-3$, $1 \leq i \leq n$.

Next we check the radio mean condition for h.

Case a: Take the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left| \frac{h(x_i) + h(x_j)}{2} \right| \geq 1 + \left| \frac{6n + i + j - 6}{2} \right| \geq n + 2 = 1 + diam(P_n \odot \overline{K}_2)$$

Case b: Take the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(y_i, y_j) + \left| \frac{h(y_i) + h(y_j)}{2} \right| \geq 3 + \left| \frac{2n + i + j - 6}{2} \right| \geq n + 2$$

Case c: Take the pair (z_i, z_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(z_i, z_j) + \left| \frac{h(z_i) + h(z_j)}{2} \right| \geq 3 + \left| \frac{4n + i + j - 6}{2} \right| \geq n + 2$$

Case d: Take the pair (y_i, x_j), $1 \leq i, j \leq n$

$$d(y_i, x_j) + \left| \frac{h(y_i) + h(x_j)}{2} \right| \geq 1 + \left| \frac{4n + i + j - 6}{2} \right| \geq n + 2$$

Case e: Take the pair (z_i, x_j), $1 \leq i, j \leq n$

$$d(z_i, x_j) + \left| \frac{h(z_i) + h(x_j)}{2} \right| \geq 1 + \left| \frac{5n + i + j - 6}{2} \right| \geq n + 2$$

Case f: Take the pair (y_i, z_j), $1 \leq i, j \leq n$

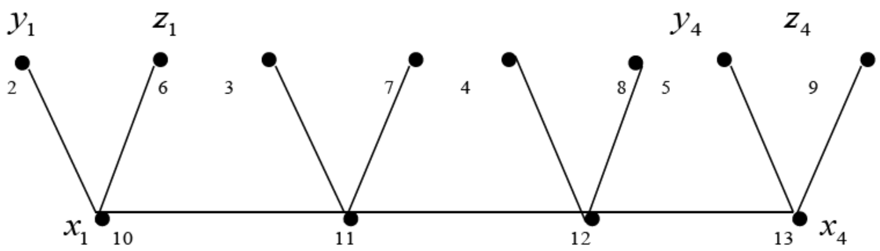
$$d(y_i, z_j) + \left| \frac{h(y_i) + h(z_j)}{2} \right| \geq 2 + \left| \frac{3n + i + j - 6}{2} \right| \geq n + 2$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of $P_n \odot \overline{K}_2$. Therefore $rmn(P_n \odot \overline{K}_2) \leq rmn(h) = 4n - 3$

Since h is injective, $rmn(P_n \odot \overline{K}_2) \geq 4n - 3$ for all radio mean labelings h and hence

$rmn(P_n \odot \overline{K}_2) = 4n - 3$, $n \geq 3$.

Example 2.2



$rmn(P_4 \square\ \overline{K_2}) = 13$

Figure 2

3. Main Results for cycle related graph

Theorem 3.1 $rmn(K_n \odot \overline{K_2}) = 3n,\ n \geq 2.$

Proof: Let $x_1, x_2, \ldots, x_n$ be the vertices of the complete graph K_n.

For $1 \leq i \leq n$, let y_i, z_i be the vertices of i^{th} copy of $\overline{K_2}$, which are adjacent to x_i.

The resultant graph is $K_n \odot \overline{K_2}$) whose edge set is

$E = \{x_n x_1, x_i x_{i+1} / 1 \leq i \leq n - 1\} \cup \{x_i y_i, x_i z_i / 1 \leq i \leq n\}$ and $diam(K_n \odot \overline{K_2}) = 3$.

Define h: $V(K_n \odot \overline{K_2}) \rightarrow N$ by

$h(x_i) = 2n + i,\ 1 \leq i \leq n$

$h(y_i) = 2i - 1,\ 1 \leq i \leq n$

$h(z_i) = 2i,\ 1 \leq i \leq n$

Next we check the radio mean condition for h.

Case a: Take the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left| \frac{h(x_i) + h(x_j)}{2} \right| \geq 1 + \left| \frac{4n + i + j}{2} \right| \geq 4 = 1 + diam(K_n \odot \overline{K_2})$$

Case b: Take the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(y_i, y_j) + \left| \frac{h(y_i) + h(y_j)}{2} \right| \geq 3 + \left| \frac{2i + 2j - 2}{2} \right| \geq 4$$

Case c: Take the pair (z_i, z_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(z_i, z_j) + \left|\frac{h(z_i)+h(z_j)}{2}\right| \geq 3 + \left|\frac{2i+2j}{2}\right| \geq 4$$

Case d: Take the pair (y_i, x_j), $1\leq i, j\leq n$

$$d(y_i, x_j) + \left|\frac{h(y_i)+h(x_j)}{2}\right| \geq 1 + \left|\frac{2n+2i+j-1}{2}\right| \geq 4$$

Case e: Take the pair (z_i, x_j), $1\leq i, j\leq n$

$$d(z_i, x_j) + \left|\frac{h(z_i)+h(x_j)}{2}\right| \geq 1 + \left|\frac{2n+2i+j}{2}\right| \geq 4$$

Case f: Take the pair (y_i, z_j), $1\leq i, j\leq n$

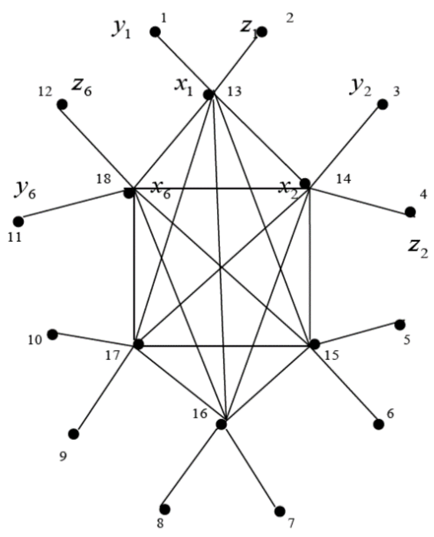
$$d(y_i, z_j) + \left|\frac{h(y_i)+h(z_j)}{2}\right| \geq 2 + \left|\frac{2i+2j-1}{2}\right| \geq 4$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of $K_n \odot \overline{K_2}$). Therefore $rmn(K_n \odot \overline{K_2}) \leq rmn(h) = 3n$

Since h is injective, $rmn(K_n \odot \overline{K_2}) \geq 3n$ for all radio mean labelings h and hence

$rmn(K_n \odot \overline{K_2}) = 3n$, $n \geq 2$.

Example 3.1



$rmn(K_6 \odot \overline{K_2}) = 18$

Figure 3

Theorem 3.2 rmn($W_n \odot \overline{K_2}$) =3n + 1, n ≥ 3.

Proof. Let u, $x_1, x_2, ..., x_n$ be the vertices of the wheel W_n and let y_i, z_i be the vertices of $\overline{K}_2$ which are joined to the vertex x_i of the wheel W_n, $1 \le i \le n$. The resultant graph is

$W_n \odot \overline{K_2}$) whose edge set is E = $\{x_i x_{i+1}, x_n x_1 / 1 \le i \le n-1\} \cup \{x_i y_i, x_i z_i, ux_i / 1 \le i \le n\}$

Clearly diam($W_n \odot \overline{K_2}$) = 4. Define a function h: V($W_n \odot \overline{K_2}$)$\rightarrow$N by

$h(x_i) = 2n+i,\ 1 \le i \le n$

$h(y_i) = i,\ 1 \le i \le n;$

$h(z_i) = n+i,\ 1 \le i \le n;$

$h(u) = 3n+1$

Next we check the radio mean condition for h.

Case a: Take the pair (x_i, x_j), $i \neq j$, $1 \le i, j \le n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i)+h(x_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{4n+i+j}{2} \right\rceil \ge 5 = 1 + \text{diam}(W_n \odot \overline{K_2})$$

Case b: Take the pair (y_i, y_j), $i \neq j$, $1 \le i, j \le n$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i)+h(y_j)}{2} \right\rceil \ge 3 + \left\lceil \frac{i+j}{2} \right\rceil \ge 5$$

Case c: Take the pair (z_i, z_j), $i \neq j$, $1 \le i, j \le n$

$$d(z_i, z_j) + \left\lceil \frac{h(z_i)+h(z_j)}{2} \right\rceil \ge 3 + \left\lceil \frac{2n+i+j}{2} \right\rceil \ge 5$$

Case d: Take the pair (y_i, x_j), $1 \le i, j \le n$

$$d(y_i, x_j) + \left\lceil \frac{h(y_i)+h(x_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n+i+j}{2} \right\rceil \ge 5$$

Case e: Take the pair (z_i, x_j), $1 \le i, j \le n$

$$d(z_i, x_j) + \left\lceil \frac{h(z_i)+h(x_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{3n+i+j}{2} \right\rceil \ge 5$$

Case f: Take the pair (y_i, z_j), $1 \le i, j \le n$

$$d(y_i, z_j) + \left\lceil \frac{h(y_i)+h(z_j)}{2} \right\rceil \ge 2 + \left\lceil \frac{n+i+j}{2} \right\rceil \ge 5$$

Case g: Take the pair (u, x_i), $1 \le i, j \le n$

$$d(u, x_i) + \left|\frac{h(u)+h(x_i)}{2}\right| \geq 1 + \left|\frac{5n+i+1}{2}\right| \geq 5$$

Case h: Take the pair (u, y_i), $1 \leq i, j \leq n$

$$d(u, y_i) + \left|\frac{h(u)+h(y_i)}{2}\right| \geq 2 + \left|\frac{3n+i+1}{2}\right| \geq 5$$

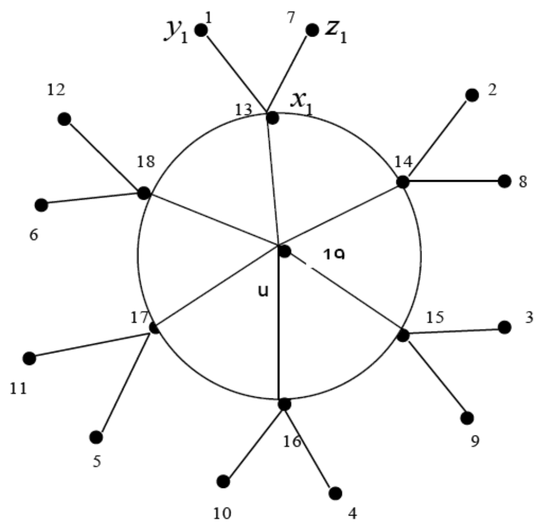
Case i: Take the pair (u, z_i), $1 \leq i, j \leq n$

$$d(u, z_i) + \left|\frac{h(u)+h(z_i)}{2}\right| \geq 2 + \left|\frac{4n+i+1}{2}\right| \geq 5$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of $W_n \odot \overline{K_2}$). Therefore $rmn(W_n \odot \overline{K_2}) \leq rmn(h) = 3n + 1$

Since h is injective, $rmn(W_n \odot \overline{K_2}) \geq 3n + 1$ for all radio mean labelings h and hence $rmn(W_n \odot \overline{K_2}) = 3n + 1$, $n \geq 3$.

Example 3.2



$rmn(W_6 \odot \overline{K_2})) = 19$

Figure 4

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