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Edge-Odd Graceful Labeling for Sum of a Path and a Finite Path

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Abstract

Choudum and Kishore [1999] found graceful labelling of the union of paths and cycles. Kaneria et. al. [2014b] found that complete bipartite graphs are graceful. Kaneria et. al. [2014c] got graceful labeling for open star of graphs. Liu et.al. [2012] investigated gracefulness of cartesian product of graphs, like as paths, cycles, and stars. A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by f+(x) ≡ Σf(x, y) (mod 2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. This article, the Edge-odd gracefulness of Pm + Pn for m = 2, 3, 4, 5, and 6 is obtained.

Key words: Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph

Section 1: Introduction

Barrientos [2005] obtained that unions of cycles and complete bipartite graphs are graceful. Gao [2007] got odd graceful labelings of some union graphs. Hoede and Kuiper [1987] proved that all wheels are graceful. Kaneria et. al. [2014a] showed that some star related graphs are graceful. Mishra and Panigrahi [2005] analyzed graceful lobsters obtained by component moving of diameter four trees. Rahim and Slamin proved that most wheel related graphs are not vertex magic. Seoud and Abdel-Aal [2013] obtained some odd graceful graphs. Sudha and Kanniga [2013] initiated some gracefulness of some new class of graphs. Vaidya and Lekha [2010] investigated odd graceful labeling of some new graphs. They also [2010] found new families of odd graceful graphs. Vaidya and Shah [2013] got graceful and odd graceful labeling of some graphs. **In this paper, the edge-odd graceful labelings of graphs as the sum of a path with n vertices and each path with 2, 3, 4, 5, and 6 vertices.**

Section 2: Edge-odd Gracefulness of the graph $P_2 + P_n$

The following definitions are first given.

Definition 2.1: Graceful graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label $|f(u) - f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2k-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Definition 2.3: $P_2 + N_n$ is a connected graph such that every vertex of P_2 is adjacent to every vertex of null graph N_n together with adjacency in both P_2 and P_n. It has n + 2 vertices and 3n edges.

Theorem 3.4: The connected graph $P_2 + P_n$ is edge – odd graceful.

Proof: The figure 1 is connected graph $P_2 + P_n$ with n + 2 vertices and 3n edges, with some arbitrary labeling to its vertices and edges as follows.

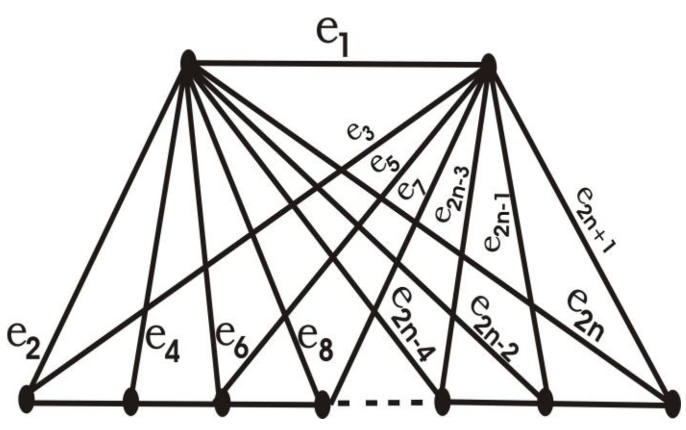

Figure 1: Edge – odd graceful graph $P_2 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

Case (i). n is odd

$f(e_i) = (2i-1)$, for $i = 1, 2, \ldots, (3n)$ … (Rule 1).

Case (ii). n is even and $i \neq 6$

$f(e_i) = (2i-1)$, for $i = 1, 2, \ldots, (2n+1)$.
$f(e_{3n-i}) = f(e_{2n+1}) + 2i + 2$, for $i = 0, 1, 2, \ldots, (n-2)$.

Define $f^+: V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by $f^+(v) \equiv \Sigma f(uv) \bmod (2k)$,
where this sum run over all edges through v (Rule 2).

Hence the map f and the induced map f^+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in $\{0, 1, 2, \ldots, (2q-1)\}$. Hence the graph $P_2 + P_n$ is edge-odd graceful.

Lemma 3.1: The connected graph $P_2 + P_6$ is edge – odd graceful.
Proof: The following graph in figure 2 is a connected graph with 8 vertices and 18 edges with some arbitrary distinct labeling to its vertices and edges.

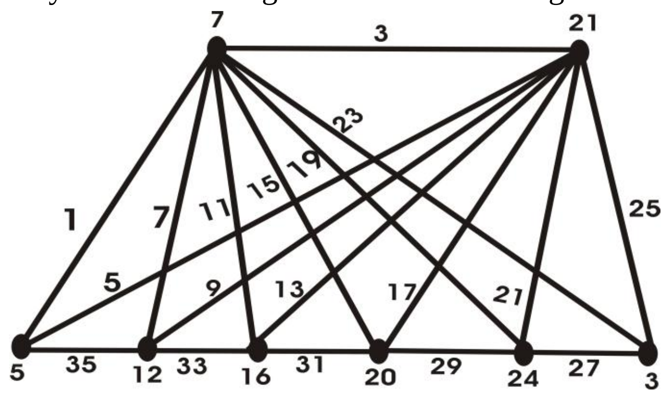

Figure 2: Edge – odd graceful graph $P_2 + P_6$

Section 4: Edge-odd gracefulness of the graph $P_3 + P_n$
Definition 4.1: $P_3 + N_n$ is a connected graph such that every vertex of K_3 is adjacent to every vertex of null graph N_n together with adjacency in both P_3 and P_n. It has n + 3 vertices and 4n+1 edges.

Theorem 4.2: The connected graph $P_3 + P_n$ for $n = 1, 2, \ldots, (4n + 1)$ is edge – odd graceful.
Proof: The figure 3 is connected graph $P_3 + P_n$ with n + 3 vertices and 4n+1 edges, with some arbitrary labeling to its vertices and edges **for case (i)**.
Case (i): $n = 1, 2, \ldots, (4n + 1)$ and $n \neq 8, 14, 20, 26, \ldots (6m + 2)$

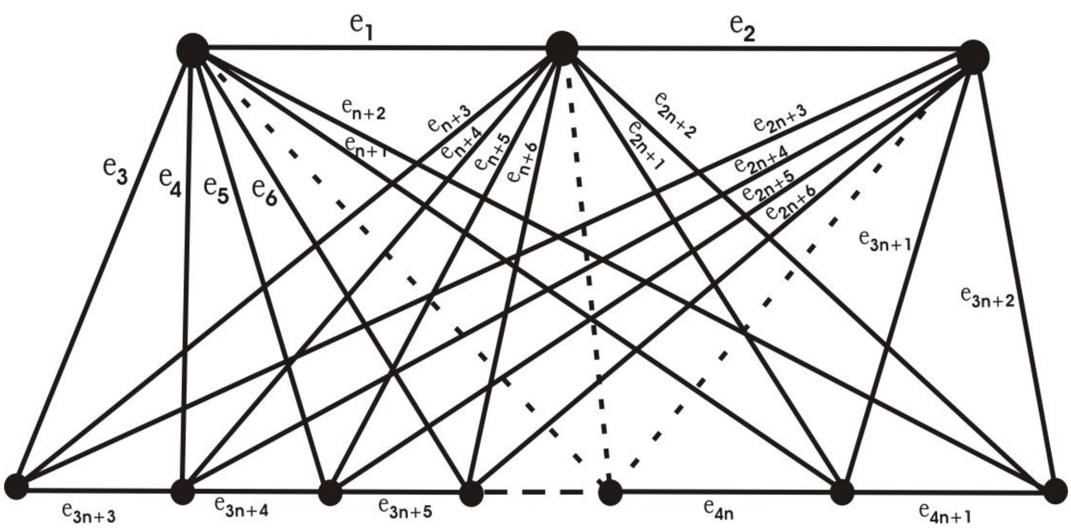

Figure 3: Edge – odd graceful graph $P_3 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

Case (i). n ≡ 0 (mod 6)

$f(e_i) = (2i-1)$, for i = 3, 4, 5, …, (4n+1)

$f(e_1) = 3$; $f(e_2) = 1$

Case (ii). n ≡ 1 (mod 6)

$f(e_i) = (2i-1)$, for i = 1, 4, 5, 6, …, (4n+1)

$f(e_2) = 5$; $f(e_3) = 3$ (Rule 3)

Case (iii). n ≡ 3, 5 (mod 6)

$f(e_i) = (2i-1)$, for i = 2, 4, 5, 6, …, (4n+1)

$f(e_1) = 5$; $f(e_3) = 1$

Case (iii). n ≡ 4 (mod 6)

$f(e_i) = (2i-1)$, for i = 2, 3, …, (4n)

$f(e_1) = 2q – 1$; $f(e_{4n+1}) = 1$

Case (iv): n ≠ 8, 14, 20, 26, … (6m + 2), m = 1, 2, …,

The figure 4 is connected graph $P_3 + P_n$ with (n + 3) vertices and (4n + 1) edges, with some arbitrary labeling to its vertices and edges **for this case (iv).**

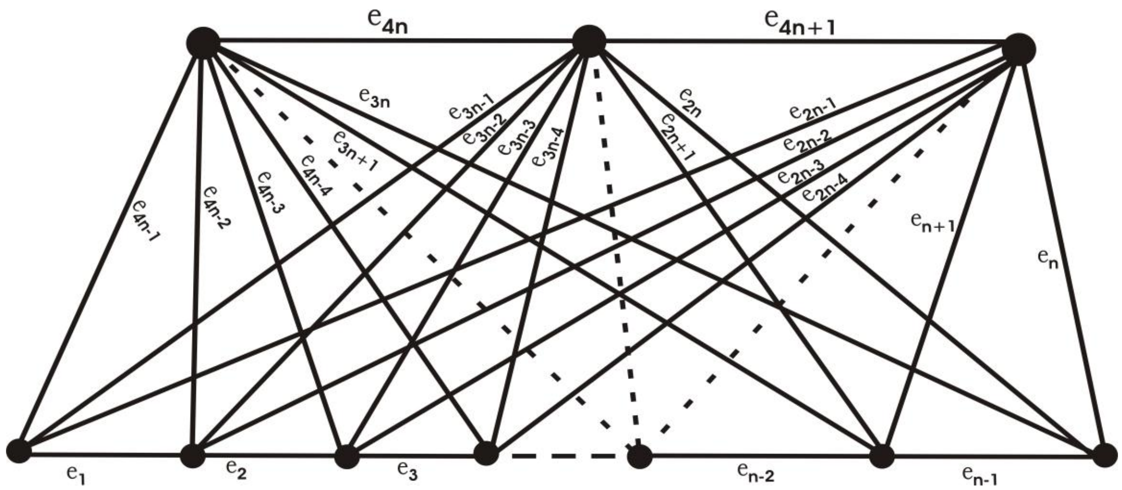

Figure 4: Edge – odd graceful graph $P_3 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by
n ≡ 2 (mod 6) (Rule 5)
$f(e_i) = (2i-1)$, for i = 1, 2, …, (n-1), (n+1), …, (3n-1), (3n+1), …, (4n+1)
$f(e_n) = 6n – 1$; $f(e_{3n}) = 2n – 1$.

Define f^+: V(G) → {0, 1, 2, …, (2q-1)} by
$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v (Rule 6).

Hence the map f and the induced map f^+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in {0, 1, 2,…., (2q -1)}. Hence the graph $P_3 + P_n$ is edge-odd graceful.

Lemma 4.3: The connected graph $P_3 + P_3$ is edge – odd graceful.
Proof: The following graph in figure 5 is a connected graph with 6 vertices and 13 edges with some arbitrary distinct labeling to its vertices and edges.

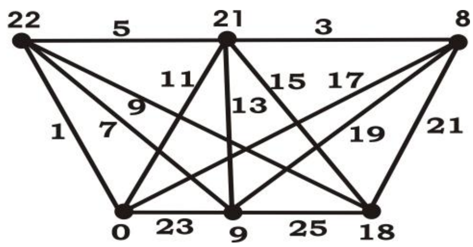

Figure 5: Edge – odd graceful graph $P_3 + P_3$

Lemma 4.4: The connected graph $P_3 + P_4$ is edge – odd graceful.
The following graph in figure 6 is a connected graph with 7 vertices and 17 edges with some arbitrary distinct labeling to its vertices and edges.

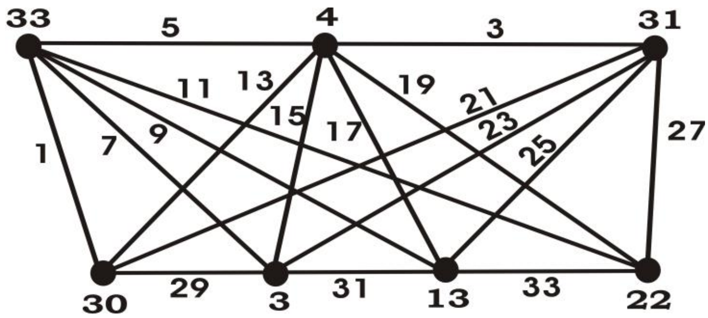

Figure 6: Edge – odd graceful graph $P_3 + P_4$

Lemma 4.4: The connected graph $P_3 + P_5$ is edge – odd graceful.
The following graph in figure 7 is a connected graph with 8 vertices and 21 edges with some arbitrary distinct labeling to its vertices and edges.

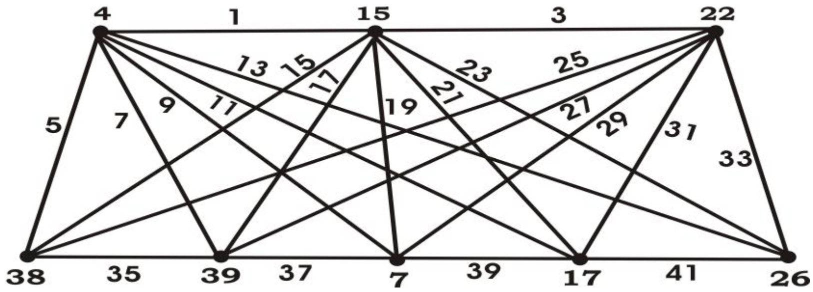

Figure 7: Edge – odd graceful graph $P_3 + P_5$

Section 5: Edge-odd Gracefulness of the graph $P_4 + P_n$

Definition 5.1: $P_4 + N_n$ is a connected graph such that every vertex of P_4 is adjacent to every vertex of null graph N_n together with adjacency in both P_4 and P_n. It has n + 4 vertices and 5n+2 edges.

Theorem 5.2: The connected graph $P_4 + P_n$ for n = 1, 2, 3, 4, …, (5n + 2) is edge – odd graceful.

Proof: The figure 8 is connected graph $P_4 + P_n$ with n + 4 vertices and 5n+2 edges, with some arbitrary labeling to its vertices and edges **for case (i). .**

Case (i): n = 1, 2, …, (5n + 2) and n ≠ 8, 14, 20, 26, … (6m + 2).

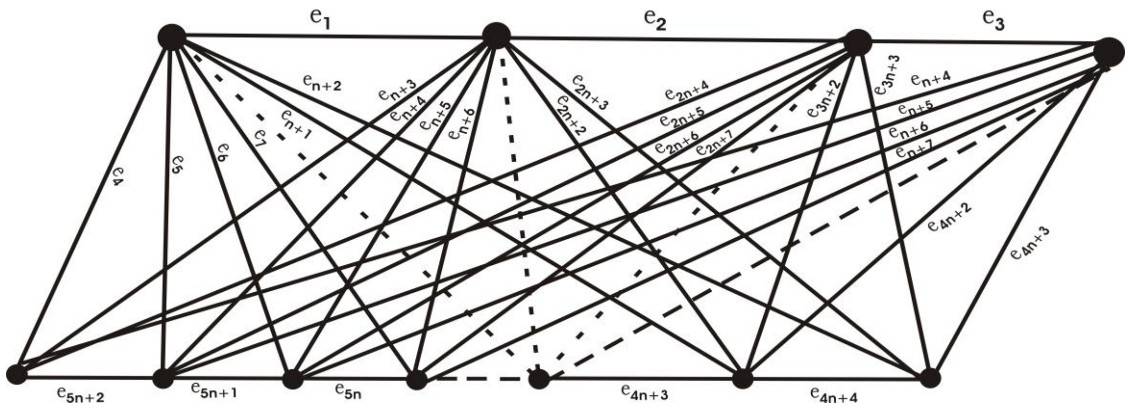

Figure 8: Edge – odd graceful graph $P_4 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

Case (i). n ≡ 0 (mod 6)

$f(e_i) = (2i-1)$, for i = 4, 5, …, (5n+2)

$f(e_1) = 3$; $f(e_2) = 5$; $f(e_3) = 1$

Case (ii). n ≡ 1(mod 6)

$f(e_i) = (2i-1)$, for i = 1, 2, …, (5n+2)

Case (iii). n ≡ 3 (mod 6)

$f(e_i) = (2i-1)$, for i = 1, 2, …, (5n+2); i ≠ 4 & 4n + 3 (Rule 7)

$f(e_4) = 8n + 5$; $f(e_{4n} + 3) = 7$

Case (iv). n ≡ 4 (mod 6)

$f(e_i) = (2i-1)$, for i = 4, 5, 6, …, (5n+2)

$f(e_1) = 5$; $f(e_2) = 1$; $f(e_3) = 3$

Case (v). n ≡ 5 (mod 6)

$f(e_i) = (2i-1)$, for i = 1, 2, 5, 6, …, (5n+2)

$f(e_3) = 7$; $f(e_4) = 5$

Case (v): n ≡ 8, 14, 20, 26, … (6m + 2), m = 1, 2, …,

The figure 9 is connected graph $P_4 + P_n$ with n + 4 vertices and 5n+2 edges, with some arbitrary labeling to its vertices and edges **for this case (v)**. .

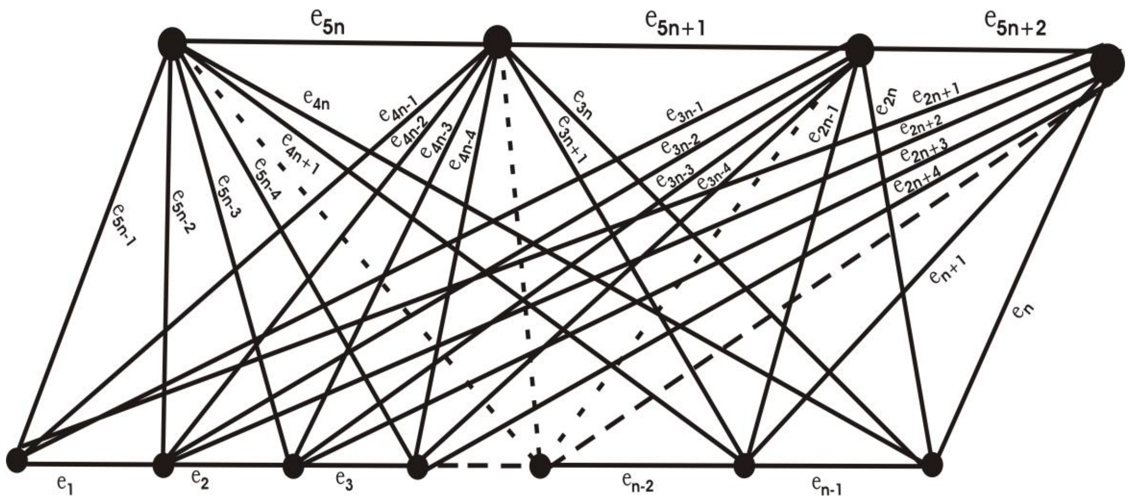

Figure 9: Edge – odd graceful graph $P_4 + P_n$

Define f: $E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ by

$n \equiv 2 \pmod 6$ (Rule 8)

$f(e_i) = (2i-1)$, for i = 1, 2, …, (5n+2); $i \neq 2n$ & $4n-1$

$f(e_{2n}) = 8n - 3$; $f(e_{4n-1}) = 4n - 3$

Define f^+: $V(G) \rightarrow \{0, 1, 2, \ldots, (2q-1)\}$ by

$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v (Rule 9).

Hence the map f and the induced map f^+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in {0, 1, 2,…., (2q-1)}. Hence the graph $P_4 + P_n$ is edge-odd graceful.

Lemma 5.3: The connected graph $P_4 + P_4$ is edge – odd graceful.

Proof: The following graph in figure 10 is a connected graph with 8 vertices and 22 edges with some arbitrary distinct labeling to its vertices and edges.

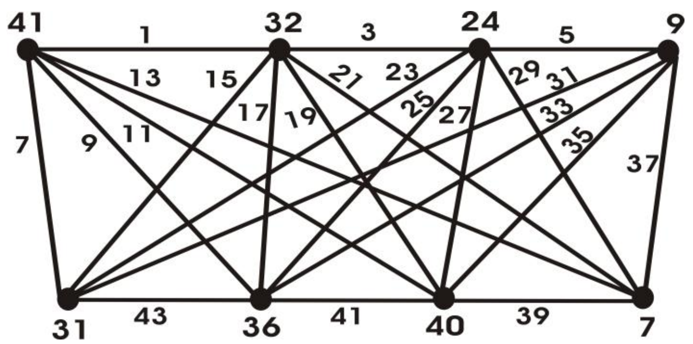

Figure 10: Edge – odd graceful graph $P_4 + P_4$

Lemma 5.4: The connected graph $P_4 + P_5$ is edge – odd graceful.

The following graph in figure 11 is a connected graph with 9 vertices and 27 edges with some arbitrary distinct labeling to its vertices and edges.

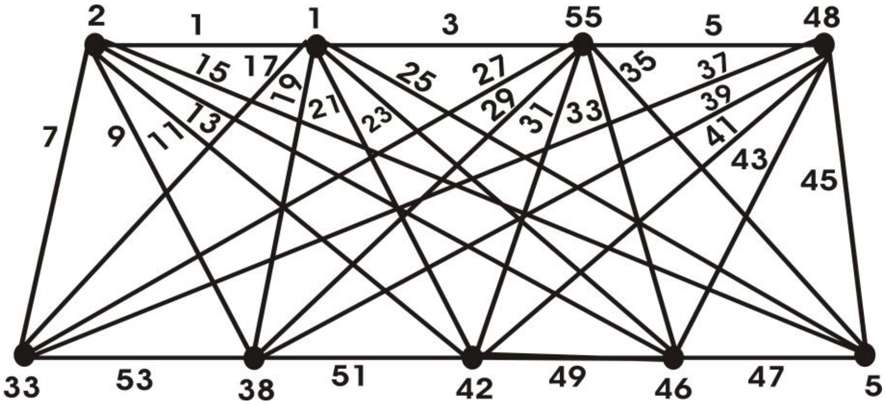

Figure 11: Edge – odd graceful graph $P_4 + P_5$

Section 6: Edge-odd gracefulness of the graph $P_5 + P_n$

Definition 6.1: $P_5 + N_n$ is a connected graph such that every vertex of P_5 is adjacent to every vertex of null graph N_n together with adjacency in both P_5 and P_n. It has n + 5 vertices and 6n+3 edges.

Theorem 6.2: The connected graph $P_5 + P_n$ for all $n \neq 7$is edge – odd graceful.

Proof: The figure 12 is connected graph $P_5 + P_n$ with n + 5 vertices and 6n+3 edges, with some arbitrary labeling to its vertices and edges.

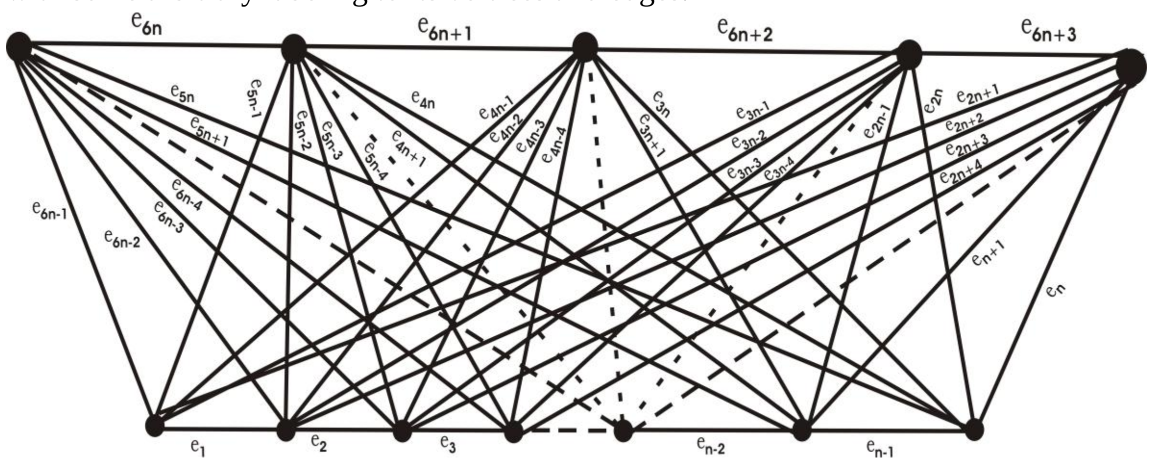

Figure 12: Edge – odd graceful graph $P_5 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

$f(e_i) = (2i-1)$, for i = 1, 2, …, (6n+3) [Rule 10]

Define $f^+: V(G) \rightarrow \{0, 1, 2, \ldots, (2q-1)\}$ by

$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v (Rule 11)

Hence the map f and the induced map f^+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in $\{0, 1, 2, \ldots, (2q-1)\}$. Hence the graph $P_5 + P_n$ is edge-odd graceful.

Lemma 6.3: The connected graph $P_5 + P_7$ is edge – odd graceful.

The graph in figure 13 is a connected graph with 12 vertices and 45 edges with some arbitrary distinct labeling to its vertices and edges.

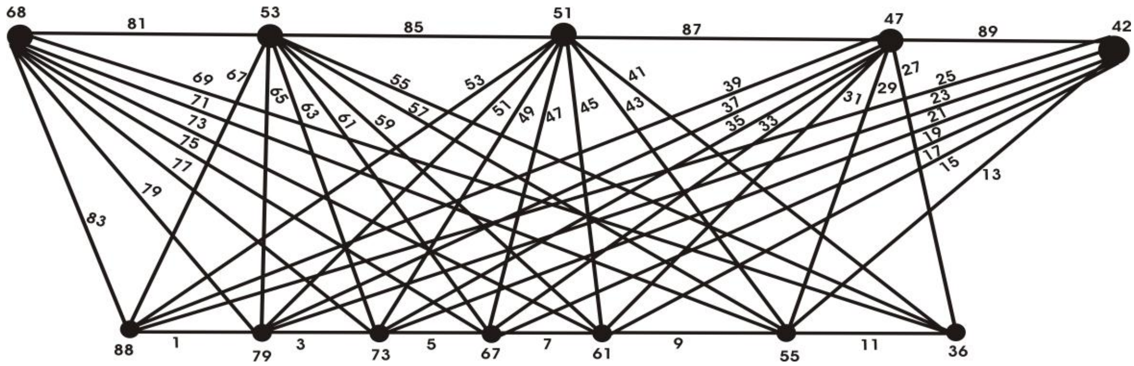

Figure 13: Edge – odd graceful graph $P_5 + P_7$

Section 7: Edge-odd Gracefulness of the graph $P_6 + P_n$

Definition 7.1: $P_6 + N_n$ is a connected graph such that every vertex of P_6 is adjacent to every vertex of null graph N_n together with adjacency in both P_6 and P_n. It has n + 6 vertices and 7n+4 edges.

Theorem 7.2: The connected graph $P_6 + P_n$ for all $n \neq 7$ and 8 is edge – odd graceful.

Proof: The figure 14 is connected graph $P_6 + P_n$ with n + 6 vertices and 7n + 4 edges, with some arbitrary labeling to its vertices and edges.

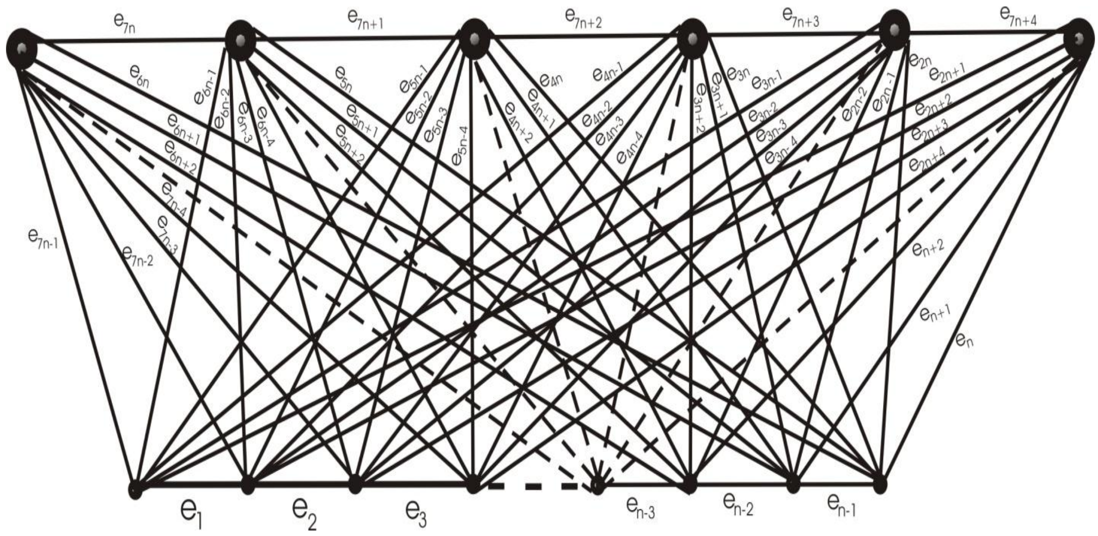

Figure 14: graph of $P_6 + P_n$

Define f: E(G) → {1, 3, …, 2q-1} by
$f(e_i) = (2i-1)$, for i = 1, 2, …, (7n+4) [Rule 12]

Define f^+: V(G) → {0, 1, 2, …, (2q-1)} by
$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v [Rule 13]

Hence the map f and the induced map f^+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in {0, 1, 2,…., (2q-1)}. Hence the graph $P_6 + P_n$ is edge-odd graceful.

Lemma 7.3: The connected graph $P_6 + P_7$ is edge – odd graceful.
The graph $P_6 + P_7$ is a connected graph with 13 vertices and 53 edges. All the edges of the graph are labeled with distinct odd numbers in such a way that there will be distinct labeling for all its vertices.
Define f: E(G) → {1, 3, …, 2q-1} by
$f(e_i) = (2i-1)$, for i = 1, 2, …, 53

Define f^+: V(G) → {0, 1, 2, …, (2q-1)} by
$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v
Hence the graph $P_6 + P_7$ is edge-odd graceful.
The graph with edge-odd graceful labeling is given in the figure 15

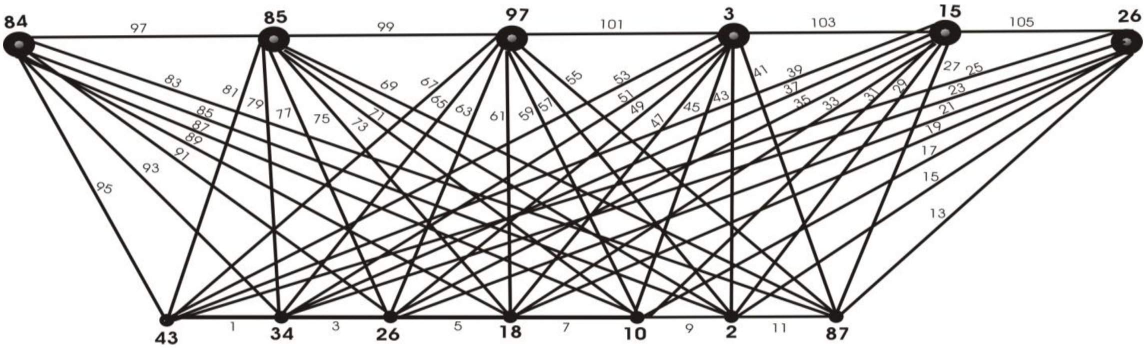

Figure 15: graph of $P_6 + P_7$

Lemma 7.4: The connected graph $P_6 + P_8$ is edge – odd graceful.

The graph $P_6 + P_8$ is a connected graph with 14 vertices and 60 edges. All the edges of the graph are labeled with distinct odd numbers in such a way that there will be distinct labeling for all its vertices.

Define f: E(G) → {1, 3, …, 2q-1} by

$f(e_i) = (2i-1)$, for i = 1, 2, …, 60

Define f^+: V(G) → {0, 1, 2, …, (2q-1)} by

$f^+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v

Hence the graph $P_6 + P_8$ is edge-odd graceful.

The graph with edge-odd graceful labeling is given in the figure 16.

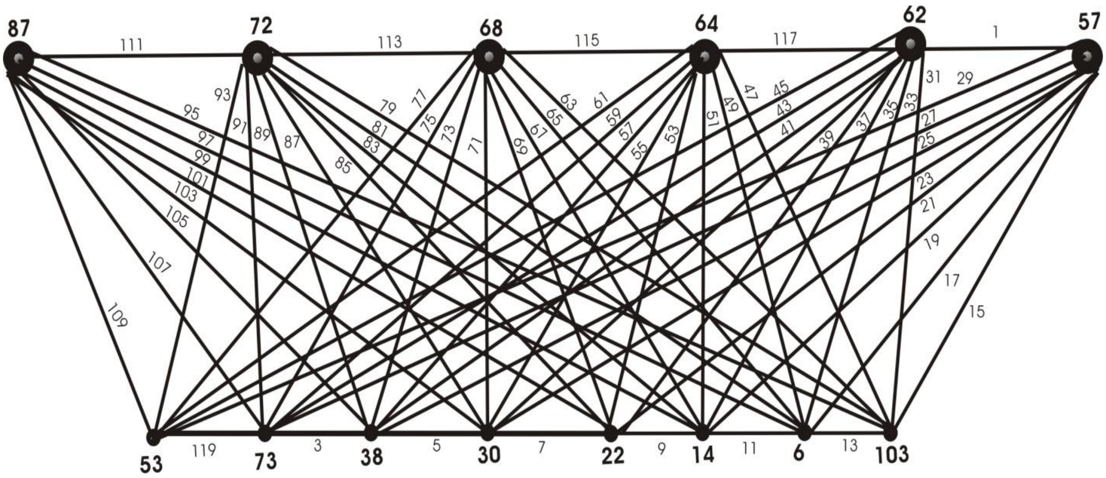

Figure 16: graph of $P_6 + P_8$

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