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$\hat{\alpha}$g Filters and $\hat{\alpha}$g Filter bases

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Abstract

In the present paper, we introduce $\hat{\alpha}$g filters and $\hat{\alpha}$g filter bases and study their properties

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1.Introduction.

Throughout the present paper (X, τ) denotes a topological space in which no separation axioms are assumed. (X, τ) will be simply denoted by X. Levine [3] initiated semiopen sets in topological spaces.Njastad [5] introduced α – open sets in general topology. Senthilkumaran etal [6] introduced $\hat{\alpha}$ generalized closed sets. In section 3, $\hat{\alpha}$g filter is defined and different $\hat{\alpha}$g filters are derived. In section 4, $\hat{\alpha}$g filters are compared. In section 6, $\hat{\alpha}$g filter base is defined and derived its properties.

2.Preliminaries

Definition 2.1: A subset A of a topological space X is said to be

1) pre open if $A \subset \text{int cl } A$ and pre closed if $\text{cl int } A \subset A$
2) regular open if A = int cl A and regular closed if A = cl int A
3) semi open if $A \subset \text{cl int } A$ and semiclosed if $\text{int cl } A \subset A$

4) α – open if $A \subset \text{int cl int } A$ and α – closed if $\text{cl int cl } A \subset A$.

Definition 2.2: A subset A of a topological space X is called $\hat{\alpha}$ generalized closed set[6] ($\hat{\alpha}$g closed)
If $\text{int cl int } A \subset U$, whenever $A \subset U$ and U is open in X.
The complement of $\hat{\alpha}$g closed set in X is $\hat{\alpha}$g open in X.
The intersection of two $\hat{\alpha}$g open sets need not be $\hat{\alpha}$g open.
In what follows, we assume finite intersection of $\hat{\alpha}$g open sets is $\hat{\alpha}$g open.

3. $\hat{\alpha}$g Filters

Definition 3.1: Let X be a topological space. A $\hat{\alpha}$g filter on X is a non empty family **F** of $\hat{\alpha}$g open subsets of X satisfying the following axioms.

1: $\phi \notin \mathbf{F}$

2: If $F \in \mathbf{F}$ and H is $\hat{\alpha}$g open such that $H \supset F$, then $H \in \mathbf{F}$

3: If $F \in \mathbf{F}$ and $H \in \mathbf{F}$, then $F \cap H \in \mathbf{F}$.

Remark 3.2:

(1) If **F** is a $\hat{\alpha}$g filter, then **F** satisfies FIP

(2) If $F \in \mathbf{F}$ then $X - F \notin \mathbf{F}$

(3) There exists no $\hat{\alpha}$g filter on ϕ.

Definition 3.3: A $\hat{\alpha}$g filter **F** on a topological space X is said to be $\hat{\alpha}$g free if and only if $\cap \{F : F \in \mathbf{F}\} = \phi$ and $\hat{\alpha}$g fixed if and only if $\cap \{F : F \in \mathbf{F}\} \neq \phi$

Definition 3.4: Let X be a topological space. Then {X} is always a $\hat{\alpha}$g filter on X called the indiscrete $\hat{\alpha}$g filter.

Theorem 3.5: Let X be a topological space and $x_0 \in X$. The family $\mathbf{F} = \{F : x_0 \in F$ and F is $\hat{\alpha}$g open$\}$ is a $\hat{\alpha}$g filter on X called the discrete $\hat{\alpha}$g filter.

Proof: As $\{X\} \in \mathbf{F}$, **F** is non empty, Since $x_0 \in F$, for every $F \in \mathbf{F}$, no member of **F** is empty and so $\phi \notin \mathbf{F}$
The verification of conditions 2 and 3 are obvious.

Theorem 3.6: Let X be a topological space and let F_0 be a non empty $\hat{\alpha}$g open subset of X.

Then $\mathbf{F} = \{F : F \supset F_0$ and F is $\hat{\alpha}$g open$\}$ is a $\hat{\alpha}$g filter on X.

Proof: Obvious

Theorem 3.7: Let X be a topological space with infinite number of elements.

Then **F** = {F: X – F is finite and F is $\hat{\alpha}g$ open} is a $\hat{\alpha}g$ filter on X called the $\hat{\alpha}g$ cofinite filter.

Proof: As X-X =ϕ is finite, X$\in$ F and hence **F** is nonempty. Let F be $\hat{\alpha}g$ open and X- F is finite. Then F is Infinite and hence no member of **F** is empty. So$\phi\notin$**F**. Let F$\in$**F** and H be $\hat{\alpha}g$ open such that H$\supset$F.
Then H$\in$ **F**.Let F, H$\in$**F** X – (F$\cap$ H) = (X-F) $\cup$ (X-H) is finite. Hence F$\cap$ H$\in$ **F**. This completes the proof.

Theorem 3.8: Let X be a topological space and let N(x) be the collection of all $\hat{\alpha}g$ open neighbourhoods of a point x$\in$X. Then N(x) is a $\hat{\alpha}g$ filter on X called the $\hat{\alpha}g$ open neighbourhood filter of X.

Proof: Obivious.

4.Comparison of $\hat{\alpha}g$ filters

Definition 4.1: Let **F** and **F'** be two $\hat{\alpha}g$ filters on the same topological space X. Then **F** is said to be finer than **F'** if and only if **F'**$\subset$ **F**. If **F**$\neq$**F'**, then **F** is said to be strictly finer than **F'**.

Theorem 4.2: Let X be a topological space with infinite number of elements and let **F** be a $\hat{\alpha}g$ filter on X such that $\cap$ {F: F$\in$**F**} = ϕ. Then **F** is finer than $\hat{\alpha}g$ cofinite filter on X.

Proof: Let **C** be the $\hat{\alpha}g$ cofinite filter on X. Let **C**$\not\subset$**F**. Then, there exists C$\in$**C** such that C$\notin$**F**. Then X – C is finite. Let X- C = {x_1, x_2, …, x_n}. Now $\cap$ {F: F$\in$**F**} = ϕ. Therefore, there exists F_i $\in$**F**(i = 1, 2, 3, …, n), such that $x_i \notin F_i$. Since **F** is a $\hat{\alpha}g$ filter, G = $\cap$ {F_i: 1$\leq$i$\leq$ n}$\in$**F**. G will not contain any element of {x_1, x_2, x_3, …x_n]. Hence X – G$\supset$ X – C. So G$\subset$ C. As **F** is a $\hat{\alpha}g$ filter C$\supset$ G implies C$\in$**F**. This is a contradiction. Hence **F** is finer than the $\hat{\alpha}g$ cofinite filter.

Theorem 4.3: Let{$\mathbf{F}_\lambda$: $\lambda\in$ $\cap$} be any nonempty family of $\hat{\alpha}g$ filters on a nonempty topological space X. Then the set**F** = $\cap${$\mathbf{F}_\lambda$: $\lambda\in$ $\cap$} is also a $\hat{\alpha}g$ filter on X.

Proof: Obvious.

5.$\hat{\alpha}g$ Filters generated by collection of sets

Theorem 5.1: Let **A** be any nonvoid family of $\hat{\alpha}g$ open subsets of a topological space X. Then, there exists a $\hat{\alpha}g$ filter on X containing **A** if and only if **A** has the FIP.
Proof: Suppose **A** has the FIP. We have to prove there exists a $\hat{\alpha}g$ filter on X

containing **A.**
Let **B** = {B: B is the intersection of a finite subfamily of **A**}
Since **A** has the FIP, no member of **B** is empty. Hence $\phi \notin$ **B.**
Let **F** = {F: F contains a member of **B** and F is $\hat{\alpha}$g open}
Evidently **F**$\supset$**A**. Let us prove **F** is a $\hat{\alpha}$g filter on X.
As $\phi \notin$ **B** and every member of **F** is a superset of some other member of **B**, it follows that $\phi \notin$ F.
The verification of other two conditions are straight forward. Hence **F** is a $\hat{\alpha}$g filter on X containing **A**. Conversely, let **F** be a $\hat{\alpha}$g filter on X containing **A**. Then **F**$\supset$**B**.
Hence a necessary condition for the existence of such a $\hat{\alpha}$g filter is that $\phi \notin$ B, that is **A** must have the FIP.

Corollary 5.2: The above $\hat{\alpha}$g filter **F** is the smallest $\hat{\alpha}$g filter of the ordered family of all $\hat{\alpha}$g filter on X containing **A**.

Definition 5.3: The $\hat{\alpha}$g filter defined in the above theorem is said to be generated by **A** and **A** is said to be $\hat{\alpha}$g sub base of **F**.

Remark 5.4: For **A** to be a sub base, it is necessary and sufficient that **A** has the FIP.

Theorem 5.5: Let **F** be a $\hat{\alpha}$g filter on a topological space X and let A be a $\hat{\alpha}$g open subset of X. Then there exists a $\hat{\alpha}$g filter **F'** finer than **F** such that A$\in$**F'** if and only if A$\cap$ F$\neq\phi$, for every F$\in$**F.**

Proof: Let A$\cap$ F$\neq\phi$, for every F$\in$**F.**
Let **A** = {A$\cap$F: F$\in$**F**}. Then **A** has F IP as follows: Let {A$\cap F_i$: $1\leq i\leq n$} be any finite subfamily of **A** and let F =$\cap$\{F_i: $1\leq i\leq n$\}.Since each F_i is a member of **F**, F is also a member of **F**.
Hence $\cap$\{A$\cap F_i$: $1\leq i\leq n$\} = A$\cap$ \{$\cap$\{F_i: $1\leq i\leq n$\}\} =A$\cap$F$\neq\phi$
Then, by the preceding theorem, there exists a $\hat{\alpha}$g filter **F'** on X containing **A**. Let us prove **F**$\subset$**F'**.
Let F$\in$**F**. A$\cap$ F $\in$**A**.As **F'** contains **A**, it follows A$\cap$ F$\in$**F'**.
Since F$\supset$ A$\cap$ F and **F'** is a $\hat{\alpha}$g filter, F$\in$**F'** Hence **F**$\subset$**F'**.
Conversely, let **F'** be a $\hat{\alpha}$g filter on X such that A$\in$**F'** and F$\subset$**F'**. Let F$\in$**F.** Hence F$\in$**F'**. As A$\in$**F'**, A$\cap$ F$\neq\phi$, **F'** being $\hat{\alpha}$g filter.

Theorem5.6: Let **F** be a $\hat{\alpha}$g filter on a topological space X and A be a $\hat{\alpha}$g open subset of X such that A$\notin$**F.**
Then X-A meets every member of **F**. Hence there exists a $\hat{\alpha}$g filter **F'** finer than **F** and not containing A.

Proof: Let $F \in \mathbf{F}$ and $X-A \cap F = \phi$. But, then $F \subset A$ and hence $A \in \mathbf{F}$, a contradiction. Hence $(X-A) \cap F \neq \phi$, that is, X-A meets every member of **F**. Then, by the preceding theorem, there exists a filter **F'** finer than **F** such that $X-A \in \mathbf{F'}$, that is $A \notin \mathbf{F'}$.

6. $\hat{\alpha g}$ Filter base

Definition 6.1: Let X be a topological space. A $\hat{\alpha g}$ filter base on X is a non empty family **B** of $\hat{\alpha g}$ open subsets of X satisfying the following axioms

i) $\phi \notin \mathbf{B}$

ii) If $F, H \in \mathbf{B}$, then there exists $G \in \mathbf{B}$ such that $G \subset F \cap H$

Remark 6.2:

i) **B** has the FIP

ii) Every $\hat{\alpha g}$ filter is a $\hat{\alpha g}$ filter base.

Theorem 6.3: The family **B** = {A} where A is any non void $\hat{\alpha g}$ open subset of a topological space X is a $\hat{\alpha g}$ filter base on X.

Proof: Straight forward.

Definition 6.4: Let X be a topological space. A nonempty collection **B**(x) of $\hat{\alpha g}$ open neighbourhoods of x is called a $\hat{\alpha g}$ base for the $\hat{\alpha g}$ open neighbourhood system of x if and only if for every $\hat{\alpha g}$ open neighbourhood N of x, there exists a $B \in \mathbf{B}$ such that $B \subset N$. We say **B**(x) is a $\hat{\alpha g}$ open local base at x.

Theorem 6.5: Let X be a topological space and let $x \in X$. Then the $\hat{\alpha g}$ open local base B(x) at x is a $\hat{\alpha g}$ filter base at x.

Proof: Obivious

Theorem 6.6: Let **B** be a family of $\hat{\alpha g}$ open subsets of a topological space X. Then **B** is a $\hat{\alpha g}$ filter base on X if and only if the family **F** consisting of all those $\hat{\alpha g}$ open subsets of X, which contain a member of **B** is a $\hat{\alpha g}$ filter on X.

Proof: **B** is nonempty if and only if **F** is non empty. Let **F** be a $\hat{\alpha g}$ filter on X. Then $\phi \notin \mathbf{F}$. As $\mathbf{B} \subset \mathbf{F}$, we have $\phi \notin \mathbf{B}$. Let $F, H \in \mathbf{B}$. Hence $F, H \in \mathbf{F}$. As **F** is a $\hat{\alpha g}$ filter, $F \cap H \in \mathbf{F}$. By the definition of **F**, there exists $G \in \mathbf{B}$ such that $G \subset F \cap H$. Hence **B** is a $\hat{\alpha g}$ filter base.

Conversely, let **B** be a $\hat{\alpha g}$ filter base.

Let $\mathbf{F} = \{F : F \supset B, B \in \mathbf{B}$ and F is $\hat{\alpha g}$ open$\}$. As **B** is a $\hat{\alpha g}$ filter base $\phi \notin \mathbf{B}$. Hence $\phi \notin \mathbf{F}$. Let $F \in \mathbf{F}$ and H be $\hat{\alpha g}$ open such that $H \supset F$. By the definition of **F**, there exists $G \in \mathbf{B}$ such that $F \supset G$. Hence $H \in \mathbf{F}$. Finally, let $F, H \in \mathbf{F}$. Then, there exists $G, K \in \mathbf{B}$ such that $F \supset G$, H$\supset$ K. As **B** is a $\hat{\alpha}$g filter base, there exists L$\in$**B** suchthat L$\subset$ G$\cap$ K.Hence L$\subset$ F$\cap$ H.This completes the proof.

Corollary6.7: Let **F** be a $\hat{\alpha}$g filter on a topological space X. Then the family **F'** which consists of those $\hat{\alpha}$g open subsets of X which contains a member of **F** is also a $\hat{\alpha}$g filter on X.

Theorem 6.8: Let **A** be any nonvoid family of $\hat{\alpha}$g open subsets of a topological space X. Then there exists a $\hat{\alpha}$g filter base on X containing **A** if and only if **A** has the FIP.

Proof: Let **A** have the FIP
Let **B** = {B: B is the intersection of finite sub family of A}. Let us show **B** is a $\hat{\alpha}$g filter base on X.

Clearly **B**$\supset$ **A.** As **A** is nonvoid, **B** is so. Since **A** has the FIP, $\phi\notin$ B.
Let F, H$\in$**B**. F$\cap$ H is the intersection of finite sub family of **A**. Hence F$\cap$ H$\in$ B.
So, **B** is a $\hat{\alpha}$g filter base.
Conversely, let **B** be a $\hat{\alpha}$g filter base on X containing **A**.$\phi\notin$ **B**.
The intersection of every finite subfamily of **B** contains a member of **B**. In particular the intersection of every finite subfamily of **A** contains a member of **B**. So, **A** has the FIP.

Remark 6.9: The $\hat{\alpha}$g filter base **B** defined in the above theorem is said to be generated by **A**.

Definition 6.10: Let **B** be a $\hat{\alpha}$g filter base on a topological space X. Then a $\hat{\alpha}$g filter **F** consisting of all those $\hat{\alpha}$g open subsets of X which contain a member of **B** is called a $\hat{\alpha}$g filter generated by **B**.
Such a $\hat{\alpha}$g filter always exists in view of theorem 6.6.

Definition 6.11: Two $\hat{\alpha}$g filter bases $\mathbf{B}_1$ And $\mathbf{B}_2$ on a topological space X are said to be equivalent if and only if they generate the same $\hat{\alpha}$g filter on X.

Definition 6.12: Let **F** be a $\hat{\alpha}$g filter on a topological space X. Then a subfamily **B** of **F** is called a $\hat{\alpha}$g base of **F** if and only if every member of **F** contains a member of **B**.

Remark 6.13:

i) A collection **B** of $\hat{\alpha}$g open subsets of a topological space X is a $\hat{\alpha}$g filter base on X if and only if **B** is a $\hat{\alpha}$g base of some $\hat{\alpha}$g filter on X.

ii) If **A** is a $\hat{\alpha}$g subbase of a $\hat{\alpha}$g filter **F**, then the family **B** of all finite intersections of members of **A** is a $\hat{\alpha}$g base for **F**.

Theorem 6.14: Let X be a topological space and N(x) be the $\hat{\alpha}$g open neighbourhood filter at x.
Then a $\hat{\alpha}$g open local base **B**(x) at x is a $\hat{\alpha}$g base of the $\hat{\alpha}$g filter N(x).

Proof: Stright forward.

Theorem 6.15: On a topological space X, a $\hat{\alpha}$g filter **F'** with $\hat{\alpha}$g base **B'** is finer than a $\hat{\alpha}$g filter **F** with $\hat{\alpha}$g base **B** if and only if every member of **B** contains a member of **B'**.

Proof: Let every member of **B** contain a member of **B'**. Let F∈**F**. Then F contains a membe of **B**, say B. By hypothesis, there exists B'∈**B'** such that B'⊂ B. Hence F⊃B'. So F∈**F'**.
Conversely, let **F**⊂**F'**. Let B∈ **B**. Then B∈ **F**⊂ **F'**. There exists B'∈**B'** such that B'⊂ B.

Theoerm6.16: Two $\hat{\alpha}$g filter bases **B** and **B'** on a topological space X are equivalent if and only if every member of **B** contains a member of **B'** and every member of **B'** contains a member of **B**.

Proof: This theorem is an immediate corollary of the preceding theorem.

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