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Inventory Model with Deteriorating Items and Time Dependent Holding Cost

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Abstract

This paper deals with the development of inventory model for deteriorating items, general demand and time dependent holding cost. In this model shortages are not allowed. Optimal total inventory cost, optimal cycle time and optimal order quantity is obtained with the help of differential calculus. Truncated Taylor's series is used for finding closed form optimal solution. Finally numerical example and sensitivity analysis is given to validate the proposed model.

Keywords: Inventory model, General demand, optimal time dependent holding cost, Deterioration

Introduction

In classical inventory models, demand rate is considered as either constant or time dependent. In case of demand rate is constant, the effects of variability of the holding cost on the total inventory cost functions of such model have also been considered. In real life, it is observed that demand for a particular product can be influenced by internal factors such as deterioration, price and variability. The main objective of inventory management is to minimize the total inventory carrying cost.

Most of the items deteriorate with time. When the inventory undergoes decay or deterioration, there is a loss of original value of commodity that results in the decreasing usefulness from the original one. Certain products such as medicine, blood, green vegetables, radioactive chemicals, volatiles decrease under deterioration during their normal storage period. Thus while determining optimal inventory policy of that type of product; the loss due to deterioration can not be ignored. Various types of inventory models for deteriorating items were discussed by Agrawal and Jaggi [1], Roy Chowdhury and Chaudhuri [2], Padmanabhan and Vrat [3], Balkhi and

Benkherouf [4] and Yang[5], Teng et el. [6] developed optimal pricing and ordering policy under permissible delay in payments by considering demand rate is a function of price. An EOQ model for perishable items with power demand and partial backlogging developed by Singh et al. [7]. Tripathy and Pradhan [8] developed an inventory model for Weibull deteriorating items with constant demand when delay in payments is allowed to retailer to settle the account against the purchases made. Liao et al. [9], Shah [10], Huang and Chung [11] developed optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. Teng et al. [12] developed an inventory model for optimal pricing and ordering policy under permissible in payments.

For certain type of inventory, the consumption rate may be influenced by the stock levels, it means that consumption rate may go up and down with the on hand stock level. Hou [13] developed an inventory model for deteriorating items with stock dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon and shown that total cost function is convex. Goh [14] developed inventory model by considering stock dependent demand rate and variable holding cost. Padmanabhan and Vrat [15] defined stock dependent consumption rate as a function of inventory level at any instant of time and developed models for non-sales environment, Sarker et al. [16] developed an order level lot size inventory model with inventory level dependent demand and deterioration level. Datta and Pal [17] presented an inventory system where the demand rate is influenced by stock level and selling price. Balkhi and Benkherouf [18] developed an inventory model for deteriorating items with stock dependent and time varying demand rate over a finite planning horizon. Other related articles on inventory model with a stock dependent demand rate have been performed by Pal et al. [19], Datta and Pal [20] and Baker and Urban [21].

In classical inventory EOQ model, holding cost is constant but in actual practice holding cost is not constant, it is a function of time or stock dependent. Tripathi [22] developed an inventory model for time varying demand and constant demand; and time dependent holding cost and constant holding cost. Alfares {23] developed an EOQ model by considering holding cost an increasing function of time spent in shortage. Ray and Chaudhuri [24] developed an inventory system with stock dependent demand rate and shortages. Goh [25] developed a model by considering holding cost variation over time as a continuous non-linear function.

2. Notations and Assumptions

The following notations are used throughout the manuscript:

K : ordering cost per order

λ : constant annual demand rate

I (t) : on-hand inventory level at time t

h (t) : time dependent holding cost of the item at time t, $h(t) = h.t$

Q : order quantity

Q* : optimal order quantity

θ : deterioration rate ($0 \le \theta < 1$)

T : cycle time
T* : optimal cycle time
TIC : total inventory cost per cycle
TIC* : optimal total inventory cost per cycle

In addition, the following assumptions are being made throughout the manuscript

1. The demand rate is constant.
2. The holding cost is time dependent i.e. h (t) = h.t.
3. Shortages are not allowed.
4. The inventory system under consideration deals with single item.
5. The planning horizon is infinite and lead time is zero.

3. Mathematical Formulations

The objective is to minimize the total inventory cost per unit time which contains two components (a) the ordering cost and (b) the holding cost per unit time. The ordering cost per unit time is k/T. The total holding cost per cycle is the integral of the product of the holding cost h(t) and inventory cost I(t) over the whole cycle T.

$$TIC = \frac{k}{T} + \frac{1}{T}\int_0^T h(t).I(t)dt = \frac{k}{T} + \frac{1}{T}\int_0^T ht.I(t)dt \tag{1}$$

The inventory under consideration is assumed to be constant demand rate with deterioration. Hence the rate of change of inventory level is governed by the following differential equation.

$$\frac{dI(t)}{dt} + \theta.I(t) = -\lambda \tag{2}$$

With the condition I(T) = 0
The solution of (2) is given by

$$I(t) = \frac{\lambda}{\theta}\left(e^{\theta(T-t)} - 1\right) \quad , 0 \le t \le T \tag{3}$$

Using (3) in (1), we obtain

$$TIC = \frac{k}{T} - \frac{h\lambda T}{2\theta} - \frac{h\lambda}{\theta^2} - \frac{h\lambda}{\theta^3 T} + \frac{h\lambda e^{\theta T}}{\theta^3 T} \tag{4}$$

The optimal solution of (4) in obtained by solving $\frac{d(TIC)}{dT} = 0$. But it is difficult to find the solution of (4) in the present form. For finding closed form solution Truncated Taylor's series is used in exponential form i.e. $e^{\theta T} \approx 1 + \theta T + \frac{\theta^2 T^2}{2} + \frac{\theta^3 T^3}{6}$. We obtain

Total inventory cost

$$TIC = \frac{\left(k + \frac{h\lambda}{\theta^3}\right)}{T} - \frac{h\lambda}{\theta^3} + \frac{h\lambda T^2}{6} \tag{5}$$

$$\frac{d(TIC)}{dT} = -\frac{\left(k + \frac{h\lambda}{\theta^3}\right)}{T^2} + \frac{h\lambda T}{3} \tag{6}$$

$$\frac{d^2(TIC)}{dT^2} = \frac{2\left(k + \frac{h\lambda}{\theta^3}\right)}{T^3} + \frac{h\lambda}{3} > 0 \tag{7}$$

Thus the optimal solution is obtained by putting

$$\frac{d(TIC)}{dT} = 0$$

We get optimal cycle time

$$T = T^* = \left\{\frac{3\left(k + \frac{h\lambda}{\theta^3}\right)}{h\lambda}\right\}^{\frac{1}{3}} \tag{8}$$

Applying the condition I (0) = Q in (3) and using Truncated Taylor's series in exponential form i.e. $e^{\theta T} \approx 1 + \theta T + \frac{\theta^2 T^2}{2} + \frac{\theta^3 T^3}{6}$ etc. We obtain

Optimal order quantity

$$Q = Q^* = \lambda T\left(1 + \frac{\theta T}{2} + \frac{\theta^2 T^2}{6}\right) \tag{9}$$

4. Numerical Example

Example: Let λ = 200 units/year, k = \$10, 000 per order, θ = 0.9, h= \$10 per unit per year

$$T = T^* = \left\{\frac{3(k + \frac{h\lambda}{\theta^3}}{h\lambda}\right\}^{\frac{1}{3}}.$$

Putting the values in above equation, we get $T^* \approx 2.67\ years$ and putting value of T^* in equation (5), we get $TIC \approx \$4405.656$

5. Sensitivity Analysis

To find sensitivity analysis, the effect of parameters 'h', 'k', 'θ', 'λ' on optimal solution, the set of values of 'h', 'k', 'θ', 'λ' are assumed to as 'h' = 20, 40, 50, 70, 100, 200, 300, 'k'= 400, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10, 000, 'θ'= 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90 and 'λ' = 20, 50, 100, 400, 600, 800, and 1000.

Meanwhile the other parameter's values follow those data mentioned above in the numerical example. The results of sensitivity analysis are given in Table 1, 2, 3 and 4.

Table: 1 Variation of optimal solution of $T = T^{*}, TIC = TIC^{*}$ and $Q = Q^{*}$ with the variation of holding cost parameter h, keeping all the parameters same as in example.

h	$T = T^{*}$	$Q = Q^{*}$	TIC = TIC*
20	2.26	1223.34975	4770.739058
40	1.99	967.185173	4845.863228
50	1.92	906.878976	4779.402380
70	1.84	840.900608	4567.294610
100	1.23	432.404409	8042.988850
200	1.69	725.372843	2560.410000
300	1.66	703.509992	856.6089500

Table: 2 Variation of optimal solution of $T = T^{*}, TIC = TIC^{*}$ and $Q = Q^{*}$ with the variation of ordering cost k, keeping all the parameters same as in example.

k	$T = T^{*}$	$Q = Q^{*}$	TIC = TIC*
400	1.67	710.752501	68.4749920
500	1.69	725.372843	127.770543
1000	1.77	785.682291	415.778614
2000	1.92	906.878976	955.880476
3000	2.05	1020.833750	1459.048730
4000	2.16	1124.001792	1933.699213
5000	2.26	1223.349752	2385.369562
6000	2.36	1328.158912	2817.915306
7000	2.44	1416.047168	3234.280348
8000	2.52	1507.617216	3636.603170
9000	2.60	1602.952000	4026.573810
10000	2.67	1689.523401	4405.660000

Table: 3 Variation of optimal solution of $T = T^*, TIC = TIC^*$ and $Q = Q^*$ with the variation of deterioration rate θ, keeping all the parameters same as in example.

θ	$T = T^*$	$Q = Q^*$	TIC = TIC*
0.60	3.07	1526.707316	155.748431
0.65	2.96	1583.483077	1465.611258
0.70	2.87	1536.701415	2930.723893
0.75	2.81	1570.233268	3138.012315
0.80	2.75	1598.666666	3671.403300
0.85	2.71	1645.567306	4082.707660
0.90	2.67	1689.523401	4405.660000

Table: 4 Variation of optimal solution of $T = T^*, TIC = TIC^*$ with the variation of λ, keeping all the parameters same as in example.

λ	$T = T^*$	$Q = Q^*$	TIC = TIC*
20	5.36	781.541171	2600.160955
50	4.00	992.000000	3318.930017
100	3.24	1255.557024	3887.654836
400	2.26	2446.699504	4770.739124
600	2.09	3172.862649	4860.351955
800	1.99	3868.740692	4845.863273
1000	1.92	4543.394880	4779.402380

Based on the results we can make the following conclusions,

1. Based on the observations found from table 1, we can conclude that with increase in holding cost parameter h, decrease in optimal cycle time $T = T^*, TIC = TIC^*$ is not uniform but $Q = Q^*$ is a concave function of h.
2. From table 2, an increase in ordering cost k results increase in optimal cycle time $T = T^*$, optimal total inventory cost $TIC = TIC^*$ and optimal order quantity $Q = Q^*$.
3. From table 3, an increase in deterioration rate θ results increase in optimal cycle time $T = T^*$, and optimal total inventory cost $TIC = TIC^*$ but alternate slight increase and decrease in optimal order quantity $Q = Q^*$.
4. From table 4, an increase in demand rate λ in decrease in optimal cycle time $T = T^*$ but increase in optimal total inventory cost $TIC = TIC^*$ and optimal order quantity $Q = Q^*$.

7. Conclusion and Future Research

In this model we developed inventory model for deteriorating items with constant demand rate and time dependent holding cost. Shortages are not allowed and cycle time is infinite. Total inventory cost (minimum) is obtained by using differential calculus. Numerical example is given for finding minimum total inventory cost. Sensitivity analysis is given finding total inventory cost, optimal order quantity and optimal cycle time and it is observed that with the increase in holding cost h, total inventory cost increases. For managerial point of view, the variation is quite sensitive with the variation of parameters. This model is valid for long lives inventories.

The model presented in this study can be extended in different ways. The model can be extended for time dependent ordering cost. This model can also extend for time dependent deteriorating items as well as shortages etc.

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