

Mathematical Analysis of Inclined Ground Heat Exchanger

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Abstract

A ground heat exchanger (GHE) is devised for extraction or injection of thermal energy from/into the ground. Bearing strong impact on GHE performance, the borehole thermal resistance is defined by the thermal properties of the Construction materials and the arrangement of flow channels of the GHEs. The ground heat exchangers (GHE) consist of pipes buried in the soil and is used for transferring heat between the soil and the heat exchanger pipes of the ground source heat pump (GSHP). An experimental study of several types of ground heat exchangers (GHEs) installed in a steel pile foundation, including double-tube, U-tube, and multi-tube GHEs, was carried out at Saga University. Water flows through the heat exchangers and exchanges heat to or from the ground. The temperature at a certain depth in the ground remains nearly constant throughout the year and the ground capacitance is regarded as a passive means of heating and cooling of buildings. To exploit effectively the heat capacity of the ground, a heat-exchanger system has to be constructed. This is usually an array of buried pipes running along the length of a building, a nearby field or buried vertically into the ground. Circulating medium (water or air) is used in summer to extract heat from the hot environment of the building and dump it to the ground and vice versa in winter. A heat pump may also be coupled to the ground heat exchanger to increase its efficiency. A heat conduction model has been established and solved mathematically to describe the temperature response in the ground caused by a single inclined line source. Heat transfer in the GHEs with multiple boreholes is then studied by superimposition of the temperature excesses resulted from individual boreholes.

1. Introduction

Increasing concentrations of greenhouse gas emissions such as carbon dioxide (CO₂), sulphur dioxide (SO₂), and nitrogen oxides (NO_x) in the atmosphere have led to potential environmental problems in recent years. The use of geothermal energy has been recognized as a possible solution for reducing emissions.

An advantage of using geothermal energy is the stability of the temperature range of the ground at tens to hundreds of meters in depth. In addition, the ground temperature is generally lower in summer and higher in winter than that of ambient air temperature.

This system reduces primary energy consumption, maintenance, and operating costs. The heat exchange performance of the geothermal heat exchange (GHE) is an important subject of GSHP system design.

Due to reduced energy consumption and maintenance costs, ground-coupled heat pump (GCHP) systems, which use the ground as a heat source/sink, have been gaining increasing popularity for space conditioning in buildings [1,2].

The efficiency of the GCHP systems is inherently higher than that of air source heat pumps because the ground maintains a relatively stable temperature throughout the year. The system is environment-friendly, producing less CO₂ emission than the conventional alternatives.

2. The GHE system

The schematic diagrams and photographs of the U-tube, double tube, and multi-tube GHEs are shown in Fig. Steel pipes, which are used as foundation pile for houses, were buried in the ground at a depth of 20 m, and used as boreholes for the GHEs.

The U-tube and multi-tube GHEs were inserted in the steel pile, and the gaps between the steel pile and tubes were backfilled with silica sand. The U-tube is a polyethylene pipe with an outer diameter of 33 mm. The multi-tube is a polyvinyl chloride pipe with an outer diameter of 20 mm as the central pipe and four polyvinyl chloride pipes with outer diameters of 25 mm placed around the central pipe.

The central pipe is the outlet tube and the four pipes around the central pipe are the inlet tubes. The outlet tube is insulated to protect the heat exchange process from the inlet tubes. In the double-tube GHE, a stainless steel pipe with an outer diameter of 139.8 mm is used as the inlet tube of the GHE and a polyvinyl chloride pipe, 48 mm in outer diameter, is installed inside the stainless steel pipe as the outlet.

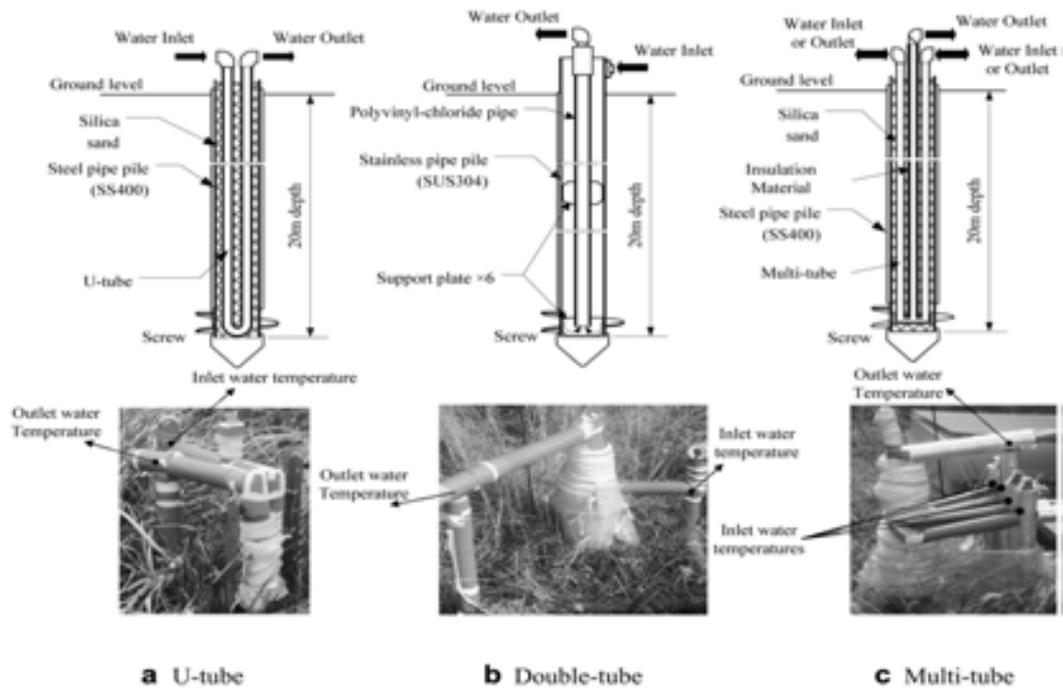


Figure-1 Akio Miyara ^{b,*}, Koutaro Tsubaki ^b, Shuntaro Inoue ^a, Kentaro Yoshida[12]

3. Analysis of Saga University

The double-tube had the highest heat exchange rate, followed by the multi-tube and U-tube GHEs. For example, the average heat exchange rate of GHEs over 24 h of continuous operation with a flow rate of 4 l/min was 49.6 W/m for the double-tube, 34.8 W/m for the multi-tube, and 30.4 W/m for the U-tube. An increasing flow rate increased the heat exchange rate of the GHEs. The heat exchange rates increased significantly for flow rate increases from 2 to 4 l/min, but only slightly changed from 4 to 8 l/min.

4. Concept of heat transfer in GHE

Heat transfer between a GHE and its surrounding soil/rock is difficult to model for the purpose of sizing the exchanger or energy analysis of the system. Besides the structural and geometrical configuration of the exchanger a lot of factors influence the exchanger performance, such as the ground temperature distribution, soil moisture content and its thermal properties, groundwater movement and possible freezing in soil. Thus, it is crucial to work out appropriate and validated tools, by which the thermal behavior of GCHP systems can be assessed and then, optimized in technical and economical aspects.

In the GHEs the heat carrier fluid flows along the borehole in one channel down to the bottom of the borehole and back upward in another channel. In cooling mode, for instance, the warm fluid induces conductive heat flow in the surrounding cooler

soil. The borehole may be conceived as a hot rod, from which heat flows to the surrounding ground.

A fundamental task for application of the GCHP technology is to grasp the heat conduction process of a single borehole in the GHE. Heat transfer in a field with multiple boreholes may be analyzed on this basis by means of the superimposition principle. The design goal is to control the temperature rise of the ground and the circulating fluid within acceptable limits over the life of the system.

Involving a time span of months or even years, the heat transfer process in the GHE is rather complicated, and should be treated, on the whole, as a transient one. Because of all the complications of this problem and its long timescale, the heat transfer process may usually be analyzed in two separated regions. One is the solid soil/rock outside the borehole, where the heat conduction has to be treated as a transient process. Since the borehole depth is much larger than its diameter, this process is often formulated by the one-dimensional line-source or cylindrical-source theory [4].

Two-dimensional model of the finite line-source [5] has also been presented by the authors to consider the axial heat flowing the ground for longer durations. Variation in load and on-off cycling of the GHE can be considered by superimposition of a series of heating pulses [6]. The temperature on the borehole wall can then be determined for any instant on specified operational conditions.

The main objective of this analysis is to determine temperatures difference of the circulating fluid in the exchanger according to the borehole wall temperature and its heat flow.

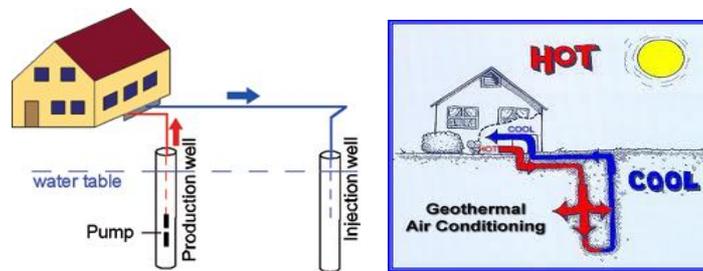
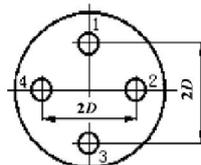


Figure-2

DIAGRAM:-

Double tube vertical ground heat exchanger.

Figure-3 from H. Y. Zeng, N. R. Diao, and Z. H. Fang [5]



Top view of double u-tube vertical ground

Heat exchange

Vertical line source of finite length for 1 & 3

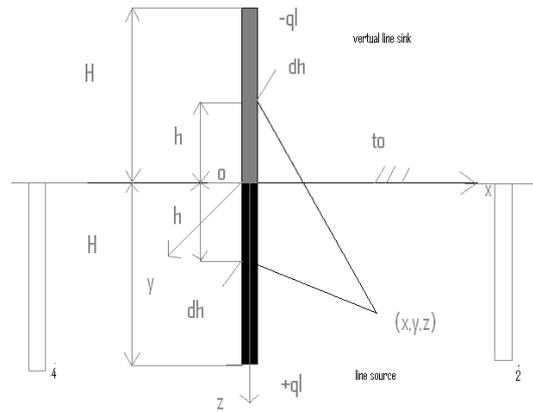


Figure-4

Vertical line source of finite length for 2 & 4.

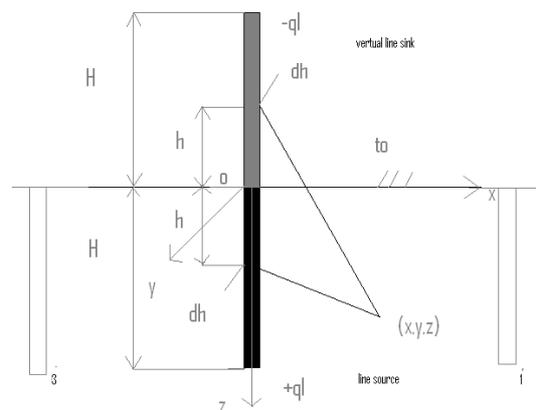


Figure-5

Where:-

H = bore hole length (m)

t_0 = temperature of ground ($^{\circ}\text{C}$)

s = half of bore hole length (m)= h

ql = heat flow per unit length of bore hole (w/m)

Mathematical analysis of inclined ground heat exchangers:-

Large no of studies and analysis are available for the vertical u tube ground heat

Set a virtual line sink with the same length H but a negative heating rate $-ql$ on symmetry to the boundary as shown in Fig. If the temperature excess is defined as $\theta = t - t_0$, the boundary condition, $\theta = 0$, is complied due to the symmetry of the line source and the virtual line sink.

Select a differential increment, dh , from the line source, which can be regarded as a point heat source. The temperature rise at the time τ in the point, m , of an infinite medium caused by this point source can be written as

$$d\theta = \left(\frac{q_l dh}{4\pi k} \right) \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{\rho^2 + (z-h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{\rho^2 + (z-h)^2}} \right\} \quad (1)$$

From H. Y. Zeng, N. R. Diao, and Z. H. Fang [5] where k and a denote the thermal conductivity and thermal diffusivity of the medium, respectively

The real solution of the temperature excess can be obtained by integrating contributions of all the increments of the line source and sink. That is,

$$d\theta = \left(\frac{q_l dh}{4\pi k} \right) \int_0^H \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{\rho^2 + (z-h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{\rho^2 + (z-h)^2}} - \frac{\operatorname{erfc} \left(\frac{\sqrt{\rho^2 + (z+h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{\rho^2 + (z+h)^2}} \right\} dh \quad (2)$$

From H. Y. Zeng, N. R. Diao, and Z. H. Fang [5]

For discrete value of temperature difference, we can put value of different parameters in above equation. so that we get particular value at particular condition of bore hole. In above condition of vertical ground heat exchanger the length of bore hole is constant throughout the analysis, but in case of inclined ground heat exchanger value of borehole length will continuously get affected due to the increment in angle.

So for the discrete value of temperature difference eq (2), can be rewritten in following way, (take $\rho = r$)

$$d\theta = \left(\frac{q_l dh}{4\pi k} \right) \int_0^H \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z-h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{r^2 + (z-h)^2}} - \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z+h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{r^2 + (z+h)^2}} \right\} dh \quad (3)$$

It can be seen from Eq. (3) that the temperature on the borehole wall, where $r = r_b$, varies with time and borehole depth. The temperature at the middle of the borehole depth ($h=0.5H$) is usually chosen as its representative temperature.

Where:-

H = active borehole depth (m)

k = ground thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)

z = axial coordinate (m)

h = variable of borehole length (m)

s = half of bore hole length = h ($0.5H$)

q = heat flow per unit length of pipe (Wm^{-1})

t = temperature

t_0 = initial and surface temperature

α = thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)

τ = time

erfc = error function

r_b = borehole radius (m) = r

DIAGRAM:-

Inclined ground heat exchanger.

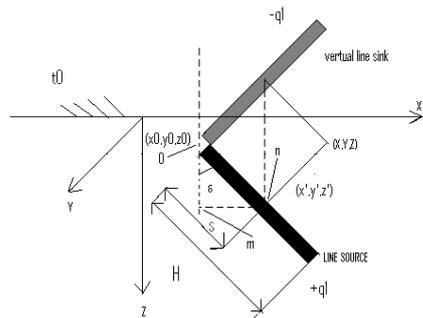


Figure-7

In Δmon :-

$$\sin \delta = (mn/on)$$

$$Mn = on * \sin \delta$$

$$Mn = s * \sin \delta$$

$$\cos \delta = (om/on)$$

$$om = on * \cos \delta$$

$$om = s * \cos \delta$$

Where:-

$$r = x' - x_0 = s \sin \delta \quad \text{and}$$

$$h = s \cos \delta$$

Then: - eq (3) will become

$$\begin{aligned} \sqrt{(r^2 + (z-h)^2)} &= \sqrt{((x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2)} \\ \sqrt{(r^2 + (z+h)^2)} &= \sqrt{((x' - x_0 - s \sin \delta)^2 + (z + s \cos \delta)^2)} \end{aligned}$$

$$\theta = \left(\frac{q_L dh}{4\pi k} \right) \int_0^H \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{((x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2)}}{2\sqrt{a\tau}} \right)}{\sqrt{(x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2}} - \frac{\operatorname{erfc} \left(\frac{\sqrt{((x' - x_0 - s \sin \delta)^2 + (z + s \cos \delta)^2)}}{2\sqrt{a\tau}} \right)}{\sqrt{(x' - x_0 - s \sin \delta)^2 + (z + s \cos \delta)^2}} \right\} dh \quad (4)$$

If we put $\delta=0$ in eq (4) then we will get eq (3), vertical ground heat exchanger eq.

$$d\theta = \left(\frac{q_L dh}{4\pi k} \right) \int_0^H \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z-h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{r^2 + (z-h)^2}} - \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z+h)^2}}{2\sqrt{a\tau}} \right)}{\sqrt{r^2 + (z+h)^2}} \right\} dh$$

Where:-

$$r = x' - x_0$$

$$h = s$$

Solution of inclined ground heat exchangers on the basis of different angles of inclinations:-.

Solution of inclined ground heat exchanger on the basis of different angles of inclinations will get some mathematical results. Those can draw on the graph paper. These results will give us behavior of temperature difference in the particular manner on the graph paper.

In this calculation, we assume all parameters are constant except the variable parameters.

Analysis in the changes of borehole length and radius of inclined ground heat exchanger. Increment in the angle δ , will affect the borehole length and radius of borehole accordingly. Before moving forward for the solution we must have study about the changes in these parameters, which will helpful in calculation.

For borehole length:-

In this diagram we assume that x axis of the borehole is 2m below from the ground (X axis) and z axis is 2m away from the ground (Z axis).

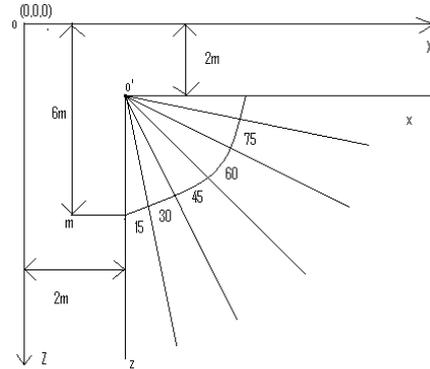


Figure-8

If we assume:-

$$6\text{m} = 100\% \text{ then } 1\text{m} = 16.66\%.$$

after inclination of 15° , this length will reduce to 5.9 m. which shows 100% will reduce to 98.33% of total length of borehole.

$$6\text{m} = 100\% \text{ then } 1\text{m} = 16.66\%.$$

$$5.9\text{m} = 16.66\% \times 5.9 = 98.33\%$$

So we can say that after increment of 15° of borehole there is reduction of 1.67% of total height.

Similarly

$$\text{At } 30^\circ, \quad 5.6\text{m} = 16.66\% \times 5.6 = 93.29\%$$

$$\text{At } 45^\circ, \quad 4.9\text{m} = 16.66\% \times 4.9 = 81.63\%$$

$$\text{At } 60^\circ, \quad 4\text{m} = 16.66\% \times 4 = 66.64\%$$

$$\text{At } 75^\circ, \quad 3.2\text{m} = 16.66\% \times 3.2 = 53.31\% \quad (5)$$

For borehole radius:-

In this diagram we assume that x axis of the borehole is 2m below from the ground (X axis) and z axis is 2m away from the ground (Z axis).

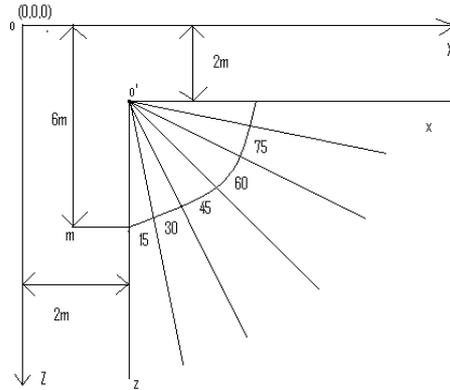


Figure-9

If we assume:-

$$2m = 100\% \text{ then } 1m = 50\%.$$

After inclination of 15° , this distance will increase to 3.3 m. which shows 100% will increase to 165% of total distance of borehole from Z axis..

$$3.3m = 50 * 3.3 = 165\%$$

So we can say that after increment of 15° of borehole there is increment in of 65% of total distance of borehole from Z axis.

Similarly

$$\text{At } 30^\circ, \quad 4.5m = 50 * 4.5 = 225\%$$

$$\text{At } 45^\circ, \quad 5.6m = 50 * 5.6 = 280\%$$

$$\text{At } 60^\circ, \quad 6.3m = 50 * 6.3 = 315\%$$

$$\text{At } 75^\circ, \quad 6.9m = 50 * 6.9 = 345\% \quad (6)$$

Solution for erfc function:-

From equation (4),

Erfc function is an error function. To solve error function following property of error function will used

$$\text{erfc}(w) = 1 - \text{erf}(w)$$

$$\text{erf}(w) = (2w/\sqrt{\pi})$$

$$\text{erfc}(w) = 1 - (2w/\sqrt{\pi})$$

From eq (4),

$$\text{erf}(w) = (2w/\sqrt{\pi})$$

$\tau = 300 \text{ sec (5 min)}$

$q_1 = 49.6 \text{ w/m (for 24 h)}$

$= 0.172 \text{ w/m (for 5 min)}$

$K = 1.1 \text{ w/mk (ground thermal conductivity of sand)}$.

angle δ	15°	30°	45°	60°	75°
θ	6.32×10^{-4}	6.07×10^{-4}	6.01×10^{-4}	5.65×10^{-4}	3.90×10^{-4}

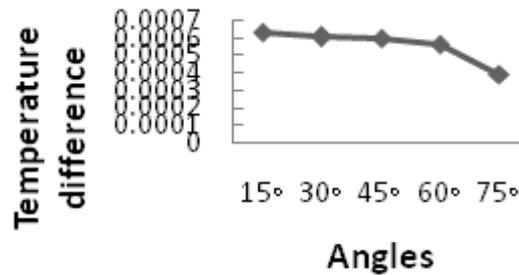


Figure-11

By Fortran language:-

Above results may be find out by using programming of Fortran language to achieve these results eq (4) have to break in 3 parts.

Part (1) - program for erf function.

$$\sqrt{((x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2)} = b$$

$$\sqrt{((x' - x_0 - s*t)^2 + (z - s*u)^2)} = b$$

Program:-

```

C    this is a test program
C    this program is written by dinesh soni
      Program test
C    this program calculates value of b
      Write (*,*) 'enter value of x, x0, s,t,z,u'
      Read (*,*) x,x0,s,t,z,u
      b= (((x'-x0-(s*t))**2+(z-(s*u))**2)**0.5
      Write (*,*) 'x=', x, 'x0=', x0, 's=', s, 't=',t, 'z=',z, 'u=',u,
              'b=', b

      Stop
      End program
    
```

$$\sqrt{((x' - x_0 - s \sin \delta)^2 + (z + s \cos \delta)^2)} = d$$

$$\sqrt{((x' - x_0 - s*t)^2 + (z - s*u)^2)} = d$$

Program:-

```
C    this is a test program
C    this program is written by dinesh soni
      Program test
C    this program calculates value of d
      Write (*,*) 'enter value of x, x0, s, t, z, u'
      Read (*,*) x, x0, s,t,z,u
      d= ((x' - x0 - (s*t)) **2 + (z - (s*u)) **2) **0.5
      Write (*,*) 'x=',x, 'x0=', x0, 's=',s, 't=',t, 'z=',z, 'u=',u,
              'd=',d
      Stop
      End program
```

Part (2) - program for erf function.

$$(2/\sqrt{\pi}) * (b / 2\sqrt{\alpha\tau}) = e1$$

Program:-

```
C    this is a test program
C    this program is written by dinesh soni
      Program test
C    this program calculates value of e1
      Write (*,*) 'enter value of b, alpha, tau'
      Read (*,*) b, alpha, tau

      e1= (b/(( alpha*tau*3.14) **0.5))
      Write (*,*) 'b=', b, 'alpha=',alpha, 'tau=',tau, 'e1=',e1
      Stop
      End program
```

$$\text{erfc}(w) = 1 - \text{erf}(w)$$

$$\begin{aligned} \text{erfc} \sqrt{((x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2) / 2\sqrt{\alpha\tau}} \\ = 1 - (2/\sqrt{\pi}) (\sqrt{((x' - x_0 - s \sin \delta)^2 + (z - s \cos \delta)^2) / 2\sqrt{\alpha\tau}}) \\ = a \end{aligned}$$

Program:-

```
C    this is a test program
C    this program is written by dinesh soni
      Program test
C    this program calculates value of e2
      Write (*,*) 'enter value of b, alpha, tau'
      Read (*,*) d, alpha, tau
      e1= (d/(( alpha*tau*3.14) **0.5))
```

```

Write (*,*) 'd=', d, 'α =', α, 'τ =', τ, 'e2=', e2
Stop
End program

```

$$\text{erfc}(w) = 1 - \text{erf}(w)$$

$$\text{erfc} \sqrt{\frac{((x' - x_0 - s \sin \delta)^2 + (z + s \cos \delta)^2)}{2\alpha\tau}}$$

$$= 1 - \frac{2}{\sqrt{\pi}} \frac{\int_0^w \exp(-\alpha\tau t^2) dt}{\sqrt{\alpha\tau}}$$

$$= c$$

Part (3) - program for temperature difference.

Program:-

```

C    this is a test program
C    this program is written by dinesh soni
C    Program test
C    this program calculate value of temperature difference
Write (*,*) 'enter value of q,k,a,b,c,d'
Read (*,*) q,k,a,b,c,d
t= ((q/(4*k*3.14))*((-a/b)+(c/d)))
Write (*,*) 'q=', q, 'k =', k, 'a=', a, 'b=', b, 'c=', c, 'd =', d, 't =', t
Stop
End program

```

Above programs will give same value as we find by mathematical calculations.

CONCLUSION

We get following results from above calculations.

in case of angle variation for inclined ground heat exchanger, we increase angle as 15°, 30°, 45°, 60°, 75°, then value of temperature difference is increases as 6.32×10^{-4} , 6.07×10^{-4} , 6.01×10^{-4} , 5.65×10^{-4} , 3.90×10^{-4} respectively. So as we increase angle, temperature difference value will increase.

so as we touches ground surface by making inclination of vertical ground heat exchanger from 15° to 75°, we will get small temperature difference. Above calculation is for only 5 minutes (300 sec) but if we put this experiment for a long period of days then it will give us considerable results.

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