Fuzzy Dot Algebras over Fuzzy Dot Fields

Dr. A. Solairaju and A. Prasanna

Associate Professor, Assistant Professor, Research Department of Mathematics
Jamal Mohamed College, Tiruchirappalli -620020, India
E-mail: Solairama@yahoo.co.in, apj_jmc@yahoo.com

Abstract

S. Nanda, N.R. Das (2010), introduced the notation of fuzzy algebras over fuzzy field. In this paper some of their basic properties are discussed in fuzzy dot Algebras over fuzzy dot field.

Introduction


Preliminaries

Definition 1.1:
A non empty set x with a constant o and a binary operation * is called a d-algebra, if it satisfies the following axioms:
(d₁) x * x = o
(d₂) 0 * x = 0
(d₃) x * y = 0 and y * x = 0, x = y
For all x, y ∈ X

Definition 1.2:
Let x be a d-algebra and I be a subset of x. Then I is called d-ideal of x if it satisfies following conditions.
(Id₁) 0 ∈ I
(Id₂) x * y ∈ I and y ∈ I => x ∈ I
(Id₃) x ∈ I and y ∈ x => x * y ∈ I

**Definition 1.3:**
A fuzzy set µ in x is called fuzzy ideal of x if it satisfies the following inequalities:
(Fd₁) µ(0) ≥ µ(x)
(Fd₂) µ(x) ≥ min {µ(x*y), µ(y)}
(Fd₃) µ(x*y) ≥ min {µ(x), µ(y)} for all x, y ∈ X

**Definition 1.4:**
A fuzzy subset µ of x is called a fuzzy dot d-ideal of x if it satisfies (Fd₁) and the following conditions.
(Fd₄) µ(x) ≥ µ(x*y) * µ(y)
(Fd₅) µ(x*y) ≥ µ(x) * µ(y) for all x, y ∈ X

Let X be a field and F a fuzzy field in X

**Definition 1.5:**
Let Y be an algebra over X and let A be a fuzzy subset of Y. Then A is called a fuzzy dot algebra in Y if for all x, y ∈ Y and λ ∈ X:
µₐ(x+y) ≥ { µₐ(x), µₐ(y) }
µₐ(λx) ≥ { µₐ(λ), µₐ(x) }
µₐ(xy) ≥ { µₐ(x), µₐ(y) }

If F is an ordinary fields (in particular F=X), then
ii. is replaced by
ii. µₐ(λx) ≥ µₐ(x)

**Definition 1.6:**
Let A be a fuzzy dot algebra in Y. Then A is called a fuzzy dot left ideal if for all x, y ∈ Y:
µₐ(xy) ≥ µₐ(y)

A is a fuzzy dot right ideal if
µₐ(xy) ≥ µₐ(y)

A is a fuzzy dot ideal if it is both a fuzzy dot left ideal and a fuzzy right ideal.
Let X be a field and F a fuzzy field in X

**Definition 1.7:**
Let Y be an algebra over X and let A be a fuzzy subset of Y. Then A is called a fuzzy dot algebra in Y if for all x, y ∈ Y and λ ∈ X:
µₐ(x+y) ≥ { µₐ(x), µₐ(y) }
µₐ(λx) ≥ { µₐ(λ), µₐ(x) }

$\mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \}$

if $F$ is an ordinary fields (in particular $F=\mathbb{X}$), then
ii. is replaced by
ii. $\mu_A(\lambda x) \geq \mu_A(x)$

**Definition 1.8:**

Let $A$ be a fuzzy dot algebra in $Y$. Then $A$ is called a fuzzy dot left ideal if for all $x, y \in Y$.

$\mu_A(xy) \geq \mu_A(y)$

$A$ is a fuzzy dot right ideal if

$\mu_A(xy) \geq \mu_A(y)$

$A$ is a fuzzy dot ideal if it is both a fuzzy dot left ideal and a fuzzy right ideal.

**Theorem 2.1**

$A$ is fuzzy dot algebra in $Y$ over $F$ if and only if for all $x, y \in Y$ and $\lambda_1, \lambda_2 \in \mathbb{X}$

i. $\mu_A(\lambda_1 x + \lambda_2 y) \geq \{ \mu_F(\lambda_1), \mu_A(x) \}. \{ \mu_F(\lambda_2), \mu_A(y) \}$

ii. $\mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \}$

$A$ is fuzzy dot ideal $F$ if and only if (i) holds and

ii. $\mu_A(xy) \geq \mu_A(y)[ \mu_A(xy) \geq \mu_A(x)]$

If $F$ is an ordinary fields (in particular if $F=\mathbb{X}$), then (i) is replaced by

$\mu_A(\lambda_1 x + \lambda_2 y) \geq \{ \mu_A(x), \mu_A(y) \}$

**Proof:**

$\mu_A(\lambda x+\lambda y) \geq \{ \mu_A(\lambda x), \mu_A(\mu y) \}$

$\geq \{ \mu_A(\lambda), \mu_A(\mu x) \}. \{ \mu_F(\mu), \mu_A(y) \}$

Conversely

$\mu_A(1.x+1.y) \leq \{ \mu_F(1), \mu_A(x) \}. \{ \mu_F(1), \mu_A(y) \}$

$\geq \mu_A(x). \mu_A(y)$

$\mu_A(\lambda x) = \mu_A(\lambda x+0.x)$

$\{\mu_F(\lambda), \mu_A(x)\}. \{ \mu_F(0), \mu_A(y) \}$

$= \{\mu_F(\lambda), \mu_A(x)\} = \mu_A(xy)$

$\geq \{ \mu_A(x). \{ \mu_F(y) \}$ follows automatically

This completes the first part of the proof.

For the second part

$\mu_A(xy) \geq \mu_A(x)$
and
\[ \mu_A(xy) \geq \mu_A(y) \]

It follows that
\[ \mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \} \]

Conversely,
\[ \mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \} \cdot \{ \mu_A(x), \mu_A(y) \} \]

So A is fuzzy dot algebra
\[ \mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \} \]

this implies \[ \mu_A(xy) \geq \mu_A(x) \]
and \[ \mu_A(xy) \geq \mu_A(y) \]

For the ideal, necessary part is obvious.
For the sufficient part,
\[ \mu_A(xy) \geq \mu_A(y) \]
implies
\[ \mu_A(xy) = \mu_A(yx) \geq \mu_A(x) \]

It given that
\[ \mu_A(xy) \geq \{ \mu_A(x), \mu_A(y) \} \]

Similarly if F is an ordinary field, then (i)is replaced by
\[ \mu_A(\lambda_1 x + \lambda_2 y) \geq \{ \mu_A(x), \mu_A(y) \} \]

This completes the proof.

**Theorem: 2.2**

Let Y and Z be algebra over a fuzzy dot fields F in a filed X and F an algebra homomorphism of Y in to Z. Let W be the fuzzy dot algebra (ideal) in Z. Then the inverse image \( f^{-1}[W] \) of W is fuzzy dot algebra (ideal) in Y.

**Proof**

for all \( x, y \in Y \) and \( \lambda_1, \lambda_2 \in X \)
\[ \mu f^{-1}[w](\lambda_1 x + \lambda_2 y) = \mu_A(\lambda_1 A(f(\lambda_1 x + \lambda_1 y))) = \mu_w(\lambda_1 f(x) + \lambda_1 f(y)) \]
\[ \{ \mu f(\lambda_1), \mu f^{-1}[w](x) \} \]
\[ \{ \mu f(\lambda_2), \mu f^{-1}[w](y) \} \]

\[ \mu f^{-1}[w](xy) = \mu_w(f(xy)) \]
\[ \mu(f(x)f(v)) \geq \{ \mu(w(f(x)), \mu_w(f(y)) \} \]
Fuzzy Dot Algebras over Fuzzy Dot Fields

\[ \{ \mu_{f^{-1}[w]}(x), \mu_{f^{-1}[w]}(y) \} \]

For ideal it is similarly proved that
\[ \mu_{f^{-1}[w]}(xy) \geq \mu_w(f(x)f(y)) \]
\[ \geq \{ \mu_w(f(x)), \mu_w(f(y)) \} \]
\[ = \{ \mu_{f^{-1}[w]}(x), \mu_{f^{-1}[w]}(y) \} \]

This completes the proof

References
