Totally Sequential Cordial Graphs

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Abstract

Suppose G= (V, E) is a simple graph with vertex set V and edge set E. A vertex labeling f: V → {0, 1} induces an edge labeling f*: E → {0, 1} defined by f*(x y) = |f(x) - f(y)|. f is called a cordial labeling of G if the number of vertices labeled ‘0’ and the number of vertices labeled ‘1’ differ by at most 1 and the number of edges labeled ‘0’ and the number of edges labeled ‘1’ differ by at most 1. A graph with a cordial labeling is called a cordial graph. If the total number of vertices and edges labeled with ‘0’ and the total number of vertices and edges labeled with ‘1’ differ by at most 1, it is called a Totally Sequential Cordial (TSC) labeling of G. A graph with a Totally Sequential Cordial labeling is called a Totally Sequential Cordial graph. In this paper we find the Totally Sequential Cordial labeling for certain graphs.

Keywords: Cordial labeling, simply sequential labeling, totally sequential cordial labeling.

2000 Mathematics Subject Classification: 05C78

Introduction

It is well known that graph theory has applications in many other fields. One area of considerable research potential on graph theory is that of graph labeling. An excellent reference on this is the survey by Gallian [6].

Two of the most important types of labelings are called graceful and harmonious labelings. Graceful labelings were introduced by Rosa [10] in 1966 and Golomb [7] in 1972. Harmonious labelings were first studied by Graham and Sloane [8] in 1980. A
third type of labeling is cordial labeling and was introduced by Cahit [3] in 1990. Recently M. Sundaram, R. Ponraj, and S. Somasundaram [12] have introduced total product cordial graphs. P.Selvaraju and B. Nirmala Gnanam Pricilla [11] have proved that $P_m \times P_n$ for all $m, n$ is a total product cordial graph. In 2002, Ibrahim Cahit [4] defined totally sequential cordial labeling, which is a weaker version of simply sequential labeling of graphs.

Suppose $G= (V, E)$ is a simple graph. A vertex labeling $f: V \rightarrow \{0, 1\}$ induces an edge labeling $f^*: E \rightarrow \{0, 1\}$ defined by $f^* (xy) = |f(x) – f(y)|$. Let $v_0$ and $v_1$ be the number of vertices labeled with ‘0’ and ‘1’ respectively. Let $e_0$ and $e_1$ be the number of edges labeled with ‘0’, and ‘1’ respectively. Such a labeling is cordial if both $|v_0 – v_1| \leq 1$ and $|e_0 – e_1| \leq 1$. A graph is called a cordial graph if it has a cordial labeling. Let $t_0= v_0 + e_0$ and $t_1 = v_1 + e_1$. If $|t_0 – t_1| \leq 1$ then the labeling is called a TSC labeling. A graph with a TSC labeling is called a TSC graph.

A.T. Diab [9] has proved that the join of the path $P_n$ and the star $K_{1, m}$ is cordial for all $n$ and all $m$ if and only if $(n, m) \neq (2, 1)$. Also he proved that the union of path $P_n$ and the star $K_{1, m}$ is cordial for all $n$ and all $m$ if and only if $(n, m) \neq (2, 1)$. But, this condition fails when $n=2$ and $m>1$ is odd. That is $P_2 + K_{1, m}$ and $P_2 \cup K_{1, m}$ where $m$ is odd are not cordial.

Cahit [8] proved that every cordial graph is TSC, $C_n$ is TSC for all $n > 2$, trees are TSC, the wheel $W_n$ is TSC for all $n > 3$. He gave some conditions for a complete graph $K_n$ to be TSC. But these conditions are contradicted by some complete graphs $K_6, K_{13}, K_{22}, K_{33}, K_{46}$ and so on.

In this paper, we prove that the graphs $C_n$, and $P_n$ are TSC graphs. Moreover, we show that the join of the path $P_n$ and the star $K_{1, m}$ is TSC if and only if $n \neq 2$ and $m$ is even; the union of the path $P_n$ and the star $K_{1, m}$ is total cordial if and only if $n \neq 2$ and $m$ is even. Also we modify the conditions for $K_n$ to be TSC, which Cahit has already established.

**Terminologies and notations**

We introduce some terminologies and notations for a graph with $4r$ vertices. Let $L_{4r}$ denote the labeling $00110011… 0011$, $S_{4r}$ denote the labeling $11001100…1100$ and $M_{2r}$ denote the labeling $0101… 0101$. Also these are modified by adding symbols at one end or other (or both). Thus $01L_{4r}$ denotes $0100110011…0011$.

For a given labeling of the graphs $G$ and $H$, let $x_i, a_i$ (for $i= 0, 1$) denote the number of vertices and edges of $G$ labeled with ‘i’ respectively. Let $y_i, b_i$ be the number of vertices and edges of $H$ labeled with ‘i’ respectively.

It follows that $v_0 = x_0 + y_0, v_1 = x_1 + y_1, e_0 = a_0 + b_0 + x_0y_0 + x_1 y_1, e_1 = a_1 + b_1 + x_0y_1 + x_1y_0$. Thus $v_0 – v_1 = (x_0 – x_1) + (y_0 – y_1)$ and $e_0 – e_1 = (a_0 – a_1) + (b_0 – b_1) + (x_0 – x_1) (y_0 – y_1)$; and $t_0 – t_1 = (v_0 – v_1) + (e_0 – e_1)$.

**Totally sequential cordial labeling for cycles and paths**

**Theorem 3.1.** The cycle graph $C_n$ is a totally sequential cordial graph for all $n>2$. 
Proof. We denote by $C_n$ the graph consisting of a cycle with $n$ points. Cahit showed that a unicyclic graph is cordial unless it is $C_{4k+2}$.

Let $n=4r+i$, where $i=0, 1, 2, 3$ and for some $r \in \mathbb{N}$. The TSC labeling we use for $C_n$ is given in the following table.

<table>
<thead>
<tr>
<th>$n=4r+i$</th>
<th>Labeling of $C_n$</th>
<th>$v_0$</th>
<th>$v_1$</th>
<th>$e_0$</th>
<th>$e_1$</th>
<th>$v_0 - v_1$</th>
<th>$e_0 - e_1$</th>
<th>$t_0 - t_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i=0$</td>
<td>$L_{4r}$</td>
<td>2r</td>
<td>2r</td>
<td>2r</td>
<td>2r</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$i=1$</td>
<td>$1L_{4r}$</td>
<td>2r</td>
<td>2r+1</td>
<td>2r+1</td>
<td>2r</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$i=2$</td>
<td>$11L_{4r}$</td>
<td>2r</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r</td>
<td>-2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$i=3$</td>
<td>$L_{4r}001$</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

The last column of this table shows that $|t_0 - t_1| = 0 \leq 1$. Hence $C_n$ is TSC for all $n>2$.

Theorem 3.2. The path graph $P_n$ is a totally sequential cordial graph for all $n$.

Proof. We denote by $P_n$ a path with $n$ points. The total sequential cordial labeling that we use are given as follows. Let $n=4r+i$, where $i=0, 1, 2, 3$ and for some natural number $r$.

<table>
<thead>
<tr>
<th>$n=4r+i$</th>
<th>Labeling of $P_n$</th>
<th>$v_0$</th>
<th>$v_1$</th>
<th>$e_0$</th>
<th>$e_1$</th>
<th>$v_0 - v_1$</th>
<th>$e_0 - e_1$</th>
<th>$t_0 - t_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i=0$</td>
<td>$L_{4r}$</td>
<td>2r</td>
<td>2r</td>
<td>2r</td>
<td>2r</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$i=1$</td>
<td>$1L_{4r}$</td>
<td>2r</td>
<td>2r+1</td>
<td>2r+1</td>
<td>2r</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$i=2$</td>
<td>$01L_{4r}$</td>
<td>2r+1</td>
<td>2r+1</td>
<td>2r</td>
<td>2r</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$i=3$</td>
<td>$001L_{4r}$</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

From the last column we observe that $|t_0 - t_1| \leq 1$. Hence $P_n$ is TSC for all $n$.

Theorem 4. The complete graph $K_4$ is not totally sequential cordial.

Proof. All possible vertex labelings satisfying the condition that $v_0 = 1$ and $v_1 = 3$ are $[0111], [1101], [1110]$.

All possible vertex labelings with $v_0 = 3$ and $v_1 = 1$ are $[0001], [0010], [0100], [1000]$.

For all the above labelings, $e_0 = 3$ and $e_1 = 3$.

Therefore $|t_0 - t_1| = 2$ not less than or equal to 1.

All possible vertex labelings with $v_0 = v_1 = 2$ are $[0011], [1001], [1100], [0110], [0101], [1010]$.

For these labelings $e_0 = 2$ and $e_1 = 4$. 
Therefore \( t_0 - t_1 = 0 - 2 = -2 \)
Hence \( K_4 \) is not totally sequential cordial.

**Join and union of Paths and Stars**

**Definition.** Let \( G_1 \) and \( G_2 \) be the graphs having disjoint point sets \( V_1 \) and \( V_2 \) and line sets \( X_1 \) and \( X_2 \) respectively. Their join denoted by \( G_1 + G_2 \) is a graph with point set \( V=V_1 \cup V_2 \) and line set \( X=X_1 \cup X_2 \) together with all lines joining \( V_1 \) and \( V_2 \).

The union of \( G_1 \) and \( G_2 \) denoted by \( G_1 \cup G_2 \) is a graph with point set \( V=V_1 \cup V_2 \) and line set \( X=X_1 \cup X_2 \).

**Theorem 5.1.** The join of the path \( P_n \) and the star \( K_{1, m} \) is TSC for all \( n \geq 2 \) and all even \( m \)

**Proof.** Let \( A_0=L_{4r}, A_1=L_{4r}0=00110011\ldots00110, A_2=L_{4r}10, A_2'=L_{4r}001, A_3=L_{4r}001, B_0=1M_{2s}=10101\ldots01, B_0'=0M_{2s}, B_1=01M_{2s} \).

Let \( n=4r+i \), where \( i=0, 1, 2, 3 \) and \( m=2s+j \), where \( j=0, 1 \) and \( r, s \in \mathbb{N} \).

<table>
<thead>
<tr>
<th>( n=4r+i, \ i=0, 1, 2, 3 )</th>
<th>Labeling of ( P_n )</th>
<th>( x_0 )</th>
<th>( x_1 )</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i=0 )</td>
<td>( A_0 )</td>
<td>( 2r )</td>
<td>( 2r )</td>
<td>( 2r )</td>
<td>( 2r-1 )</td>
</tr>
<tr>
<td>( i=1 )</td>
<td>( A_1 )</td>
<td>( 2r+1 )</td>
<td>( 2r )</td>
<td>( 2r )</td>
<td>( 2r )</td>
</tr>
<tr>
<td>( i=2 )</td>
<td>( A_2 )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
<td>( 2r )</td>
<td>( 2r+1 )</td>
</tr>
<tr>
<td>( i=2, r\neq0 )</td>
<td>( A_2' )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
<td>( 2r )</td>
</tr>
<tr>
<td>( i=2, r=0 )</td>
<td>( A_2'' )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
<td>( 2r )</td>
<td>( 2r+1 )</td>
</tr>
<tr>
<td>( i=3 )</td>
<td>( A_3 )</td>
<td>( 2r+2 )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
<td>( 2r+1 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( m=2s+j, \ j=0, 1 )</th>
<th>Labeling of ( K_{1, m} )</th>
<th>( y_0 )</th>
<th>( y_1 )</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j=0 )</td>
<td>( B_0 )</td>
<td>( s )</td>
<td>( s )</td>
<td>( s+1 )</td>
<td>( s )</td>
</tr>
<tr>
<td>( B_0' )</td>
<td>( s+1 )</td>
<td>( s+1 )</td>
<td>( s )</td>
<td>( s )</td>
<td></td>
</tr>
<tr>
<td>( j=1 )</td>
<td>( B_1 )</td>
<td>( s )</td>
<td>( s )</td>
<td>( s )</td>
<td>( s+1 )</td>
</tr>
</tbody>
</table>

By using the labelings of \( P_n \) and \( K_{1, m} \) from tables 5.1.1 & 5.1.2 and by using the formula \( v_0-v_1 = (x_0-x_1) + (y_0-y_1); \ e_0-e_1 = (a_0-a_1) + (b_0-b_1) + (x_0-x_1)(y_0-y_1). \)

Then the values of \( t_0-t_1 \) are calculated.
Table: 5.1.3 (Labelings of P_n+K_{1,m})

<table>
<thead>
<tr>
<th>n=4r+i,</th>
<th>m=2s+j,</th>
<th>Labeling of P_n</th>
<th>Labeling of K_{1,m}</th>
<th>v_0 - v_1</th>
<th>e_0 - e_1</th>
<th>t_0 - t_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=0, 1, 2, 3</td>
<td>j=0, 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i=0</td>
<td>0</td>
<td>A_0</td>
<td>B_0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>i=1</td>
<td>0</td>
<td>A_1</td>
<td>B_0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>i=2</td>
<td>0</td>
<td>A_2</td>
<td>B_0'</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>i=3</td>
<td>0</td>
<td>A_3</td>
<td>B_0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>i=0</td>
<td>1</td>
<td>A_0</td>
<td>B_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i=1</td>
<td>1</td>
<td>A_1</td>
<td>B_1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>i=2, r=0</td>
<td>1</td>
<td>A_2 or A_2'</td>
<td>B_1</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>i=2, r≠0</td>
<td>1</td>
<td>A_2'</td>
<td>B_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

From the last column of table 5.1.3, we can easily observe that |t_0 - t_1| ≤ 1, when n≠2 and m is even.

Also when n=2, all possible labelings of P_n are [0 0], [0 1], [1 1]

Labeling of K_{1,m} where m is odd is B_1. These labelings give us v_0 - v_1 = 0, e_0 - e_1 = ±2 or v_0 - v_1 = ±2, e_0 - e_1 = 0.

In either of these cases, |t_0 - t_1| =2.

Hence, P_n + K_{1,m} is TSC iff n ≠2 and m is even.

**Theorem 5.2.** The union of the path P_n and the star K_{1,m} is totally sequential cordial for all n except 2 and all even m.

**Proof.** By using the labelings of the path P_n in table 5.1.1 and the star K_{1,m} in the table 5.1.2 and using the formula v_0 - v_1 = (x_0 - x_1) + (y_0 - y_1) and e_0 - e_1 = (a_0 - a_1) + (b_0 - b_1), we can compute the values.

Table 5.2.1 (Labeling of P_n U K_{1,m})

<table>
<thead>
<tr>
<th>n=4r+i,</th>
<th>m=2s+j,</th>
<th>Labeling of P_n</th>
<th>Labeling of K_{1,m}</th>
<th>v_0 - v_1</th>
<th>e_0 - e_1</th>
<th>t_0 - t_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=0, 1, 2, 3</td>
<td>j=0, 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i=0</td>
<td>0</td>
<td>A_0</td>
<td>B_0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>i=1</td>
<td>0</td>
<td>A_1</td>
<td>B_0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i=2</td>
<td>0</td>
<td>A_2</td>
<td>B_0'</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>i=3</td>
<td>0</td>
<td>A_3</td>
<td>B_0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i=0</td>
<td>1</td>
<td>A_0</td>
<td>B_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i=1</td>
<td>1</td>
<td>A_1</td>
<td>B_1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>i=2, r=0</td>
<td>1</td>
<td>A_2 or A_2'</td>
<td>B_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i=2, r≠0</td>
<td>1</td>
<td>A_2'</td>
<td>B_1</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>i=3</td>
<td>1</td>
<td>A_3</td>
<td>B_1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
From the last column of table 5.2.1, we can easily observe that $|t_0 - t_1| \leq 1$, when $n \neq 2$ and $m$ is even.

Also when $n = 2$, all possible labelings of $P_n$ are [0 0], [0 1], [1 1].

When $m$ is odd, labelings of $K_{1, m}$ are $B_1$.

By using the labelings we compute $v_0 - v_1$, $e_0 - e_1$ and $t_0 - t_1 = \pm 2$ or $-2$.

Hence $P_n \cup K_{1, m}$ is TSC iff $n \neq 2$ and $m$ is even.

**Theorem 6:** The complete graph $K_n$ is totally sequential cordial if and only if

1. $\sqrt{n + 1}$ is an integer, when $n \equiv 0 (mod 4)$
2. $\frac{n-1}{4}$ or $\frac{n+3}{4}$ is an integer, when $n \equiv 1 (mod 4)$
3. $\sqrt{n - 1}$ or $\sqrt{n + 3}$ is an integer, when $n \equiv 2 (mod 4)$
4. $\frac{n+1}{4}$ is an integer, when $n \equiv 3 (mod 4)$

**Proof.** The complete graph $K_n$ has $n$ vertices and $nC_2$ edges. Let $f$ be a total sequential cordial labeling of $K_n$. Then $|t_0 - t_1| \leq 1$.

Also $t_0 + t_1 = n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2} = \{even if n \equiv 0, 3 (mod 4)\}$ We consider two cases.

**Case 1:** Let $t_0 + t_1$ be even.

Then $n \equiv 0, 3 (mod 4)$. Since $f$ is a TSC labeling of $K_n$, $t_0 = t_1$.

Under this labeling $f$, the complete graph $K_n$ can be decomposed as: $K_n = K_p \cup K_r \cup K_{p, r}$, where $K_p$ is the sub-complete graph of $K_n$ whose vertices are labeled with 1’s; $K_r$ is the sub-complete graph of $K_n$ whose vertices are labeled with 0’s and $K_{p, r}$ is the complete bipartite sub-graph of $K_n$ with the bipartition $V(K_p) \cup V(K_r)$ which its edges labeled with all 1’s.

Then $n = p + r$.

Clearly, for the labeling $f$, we write

$t_1 = p + rp$ and $t_0 = pC_2 + r + rC_2$.

Using $t_0 = t_1$, we get, $(r - p)^2 - 3p + r = 0$.

Put, $p = n - r$ in the above equation, we get

$4r^2 - 4(n - 1)r + n^2 - 3n = 0$

Solving this for $r$, we obtain $r_{1, 2} = \frac{(n-1)\pm \sqrt{n+1}}{2}$

This $r_{1, 2}$ will represent the order of the sub-complete graph $K_r$. We can easily see that, for $n \equiv 0 (mod 4)$, $K_n$ is TSC iff $\sqrt{n + 1}$ is an integer. Also, for $n \equiv 3 (mod 4)$, $K_n$ is TSC iff $\sqrt{n+1}$ is an integer.

**Case 2:** Let $t_0 + t_1$ be odd.

Clearly $n \equiv 1, 2 (mod 4)$

Since $f$ is a TSC labeling of $K_n$, there arise two cases $t_1 > t_0$ or $t_0 > t_1$. 
Sub case 2. (i): Assume that \( t_1 > t_0 \).

Then \( t_1 = t_0 + 1 \).

By the same decomposition of case 1, we write,
\[
t_1 = p + rp \quad \text{and} \quad t_0 = pC_2 + r + rC_2 \quad \text{__________} \quad (1)
\]
Also, \( p = n - r \quad \text{__________} \quad (2) \)

Using all these equations, we get the quadratic equation,
\[
4r^2 - 4(n - 1)r + n^2 - 3n + 2 = 0
\]

Solving this for \( r \) we get,
\[
r_{1,2} = \frac{(n - 1) \pm \sqrt{(n - 1)}}{2}
\]

This gives the order of the sub-complete graph \( k_r \). In order to have integer values for \( r_{1,2} \), for \( n \equiv 1 \pmod{4} \) and \( n \equiv 2 \pmod{4} \) respectively \( \sqrt{\frac{n - 1}{4}} \) and \( \sqrt{n - 1} \) must be an integer.

Sub case 2. (ii): Let \( t_0 > t_1 \).

We can take \( t_0 = t_1 + 1 \quad \text{__________} \quad (3) \)

By using equations (1) and (2), equation (3) becomes
\[
4r^2 - 4(n - 1)r + n^2 - 3n - 2 = 0
\]

Solving this for \( r \) we get,
\[
r_{1,2} = \frac{(n - 1) \pm \sqrt{(n + 3)}}{2}
\]

This gives the order of the sub-complete graph \( k_r \). It can easily be seen that,
when \( n \equiv 1 \pmod{4} \), \( K_n \) is TSC iff \( \sqrt{\frac{n + 3}{4}} \) is an integer and
when \( n \equiv 2 \pmod{4} \), \( K_n \) is TSC iff \( \sqrt{n + 3} \) is an integer.

Thus from case (2) we observe that
\( K_n \) is TSC iff \( \sqrt{\frac{n - 1}{4}} \) or \( \sqrt{\frac{n + 3}{4}} \) is an integer, when \( n \equiv 1 \pmod{4} \)
And for \( n \equiv 2 \pmod{4} \), \( K_n \) is TSC iff \( \sqrt{n - 1} \) or \( \sqrt{n + 3} \) is an integer.

Hence the theorem.

References


