Evaluation of Reliability of a Power Plant with the Aid of Boolean Function Expansion Algorithm

¹D. Sarada Devi, ²R. Bhuvana Vijaya and ¹A. Mallikarjuna Reddy

¹Department of Mathematics, S.K. University, Anantapur, India ²Department of Mathematics, JNTUA College of Engg., Anantapur, India

Abstract

In this paper, investigation has been carried out for the evaluation of reliability behaviour of a power plant with the aid of Boolean function expansion algorithm. The complex system under consideration consists of four power generators in a power house. The object of the system is to supply power from a power house generated by generators to critical consumers fed from an output main switch and consequently, the reliability of the supply has been calculated by considering that failure times for various components i.e., cables, generators and main switch boards etc follow arbitrary distribution.

Introduction

Nowadays, reliability is far from an abstract concept and it ranks on the same level as the performance of an equipment. Moreover, evaluation of reliability is a basic requirement for all reliability studies. However, it is an open secret that reliability evaluation becomes more complicated when complexities increase in a system. Therefore, the derivation of a symbolic reliability expression in a simplified and compact form for a complex system is of the utmost importance.

The problem of ensuring the reliability of engineering systems is extremely complex and extends to all the stages of the service life of a system. Today, there exists a large number of problems which in designing marine power plants, for example, are still solved only on the basis of logical reasoning and experience and not with the aid of reliability calculations.

Keeping the above facts in view, we, therefore, consider a complex system consisting of four generators connected in parallel. The generators G_1,G_2,G_3 & G_4 are connected with two way main switches MSB₁,MSB₂,MSB₃ and MSB₄ respectively by perfectly reliable cables. A cable C₅ connects MSB₁, MSB₂, a cable C₆ connects

MSB₂, MSB₃ and a cable C₇ connects MSB₃, MSB₄. Further, cables C₁, C₂, C₃ & C₄ connect the two way main switch MSB₁ to output main two way switch MSB₂ to output main switch OPMS₅, two way switch MSB₃ to output main switch OPMS₅ and two way switch MSB₄ to output main switch OPMS₅ respectively. Thus, the complex system under consideration is nothing but a power plant. The object of the system is to supply power generated by G₁, G₂, G₃ & G₄ to critical consumers and consequently the reliability of the power supply fed from OPMS₅ has been estimated with the aid of Boolean Function Expansion Algorithm by considering that failure times for various components of the system follow arbitrary time distribution.

Assumptions

- 1. The reliabilities of all constituent components of the system are known in advance.
- 2. The state of all components is statistically independent.
- 3. The state of each component and of the whole system is either good (operating) or bad (failed).
- 4. There is no standby or switched redundancy.
- 5. The failure times for all the components are arbitrary.
- 6. There is no repair facility.
- 7. The system can fail i.e. The supply of power can fail only if
- i. All the four generators fail.
- ii. At least one component (switch or cable) in the routes of the power supply fails.



Figure 1: System Configuration.

Notations

x_1, x_2, x_3, x_4	States of generators $G_1, G_2, G_3 \& G_4$
<i>x</i> 5, <i>x</i> 6, <i>x</i> 7, <i>x</i> 8	States of MSB ₁ ,MSB ₂ ,MSB ₃ & MSB ₄
<i>x</i> 9, <i>x</i> 10, <i>x</i> 11, <i>x</i> 12, <i>x</i> 14, <i>x</i> 15, <i>x</i> 16	States of the cables $C_1, C_2, C_3, C_4, C_5, C_6 \& C_7$
<i>X</i> ₁₃	State of OPMS ₄
x'_k	Negation of x_k (k=1-16)
\wedge	Conjunction
\vee	Disjunction
X _i	$\int 0$ in bad state
	1 in good state (i=1-16)
Pr (f=1)	The probability of the successful operation of the function
	I.

Formulation of Mathematical Model

By using Boolean Function Technique, the conditions of capability for the successful operation of the complex system in terms of logical matrix are expressed as

$$f(x_{1}x_{2}....x_{16}) = \begin{cases} x_{1} & x_{5} & x_{9} & x_{13} \\ x_{1} & x_{5} & x_{14} & x_{6} & x_{10} & x_{13} \\ x_{1} & x_{5} & x_{14} & x_{6} & x_{15} & x_{7} & x_{11} & x_{13} \\ x_{1} & x_{5} & x_{14} & x_{6} & x_{15} & x_{7} & x_{16} & x_{8} & x_{12} & x_{13} \\ x_{2} & x_{6} & x_{10} & x_{13} \\ x_{2} & x_{6} & x_{15} & x_{7} & x_{16} & x_{8} & x_{12} & x_{13} \\ x_{2} & x_{6} & x_{15} & x_{7} & x_{16} & x_{8} & x_{12} & x_{13} \\ x_{2} & x_{6} & x_{15} & x_{7} & x_{16} & x_{8} & x_{12} & x_{13} \\ x_{3} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{3} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{3} & x_{7} & x_{15} & x_{6} & x_{14} & x_{5} & x_{9} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{10} & x_{13} \\ x_{4} & x_{8} & x_{16} & x_{7} & x_{15} & x_{6} & x_{14} & x_{5} & x_{9} & x_{13} \\ x_{5} & x_{5} \\ x_{5} & x_{5} \\ x_{5} & x_{5} \\ x_{5} & x_{5} &$$

Solution of the Model

By the application of algebra of logic equation (1) may be written as

$$f(x_1 x_2, \dots, x_{16}) = x_{13} \wedge g(x_1, x_2, x_3, \dots, x_{12}, x_{14}, x_{15}, x_{16})$$
(2)

where

 $g(x_1x_2, x_3, \dots, x_{12}, x_{14}, \dots, x_{16}) =$

L								
$ x_1 $	x_5	x_9						
		x_6	x_{10}	x_{14}				
		x_6	<i>x</i> ₇	<i>x</i> ₁₁	x_{14}	<i>x</i> ₁₅		
		x_6	<i>x</i> ₇	x_8	<i>x</i> ₁₂	<i>x</i> ₁₄	<i>x</i> ₁₅	<i>x</i> ₁₆
<i>x</i> ₂	x_6	x_{10}						
		x_5	x_9	x_{14}				
		<i>x</i> ₇	<i>x</i> ₁₁	<i>x</i> ₁₅				
		<i>x</i> ₇	X_8	<i>x</i> ₁₂	<i>x</i> ₁₅	<i>x</i> ₁₆		
<i>x</i> ₃	<i>x</i> ₇	x_{11}						
		x_6	x_{10}	<i>x</i> ₁₅				
		x_5	x_6	x_9	x_{14}	<i>x</i> ₁₅		
		x_8	<i>x</i> ₁₂	x_{16}				
<i>x</i> ₄	x_8	<i>x</i> ₁₂						
		<i>x</i> ₇	<i>x</i> ₁₁	<i>x</i> ₁₆				
		x_6	<i>x</i> ₇	x_{10}	<i>x</i> ₁₅	x_{16}		
		x_5	x_6	<i>x</i> ₇	x_9	<i>x</i> ₁₄	<i>x</i> ₁₅	<i>x</i> ₁₆

Let us take x_{15} and break the complex event into incompatible events as follows: $g(x_1, x_2, \dots, x_{12}, x_{14}, x_{15}, x_{16}) = x'_{15} y_0 \lor x_{15} y_1$ (4)

where

$$y_{0} = \begin{vmatrix} x_{1} & x_{5} & x_{9} \\ & x_{6} & x_{10} & x_{14} \\ x_{2} & x_{6} & x_{10} & \\ & x_{5} & x_{9} & x_{14} \\ x_{3} & x_{7} & x_{11} & \\ & x_{8} & x_{12} & x_{16} \\ x_{4} & x_{8} & x_{12} & \\ & & x_{7} & x_{11} & x_{16} \end{vmatrix}$$
(5)

$$y_{1} = \begin{vmatrix} x_{1} & x_{5} & x_{9} \\ & x_{6} & x_{10} & x_{14} \\ & x_{6} & x_{7} & x_{11} & x_{14} \\ & x_{6} & x_{7} & x_{8} & x_{12} & x_{14} & x_{16} \\ x_{2} & x_{6} & x_{10} & & & & \\ & x_{5} & x_{9} & x_{14} & & & \\ & x_{7} & x_{11} & & & & \\ & x_{7} & x_{8} & x_{12} & x_{16} & & \\ x_{3} & x_{7} & x_{11} & & & & \\ & & x_{6} & x_{10} & & & \\ & & x_{5} & x_{6} & x_{9} & x_{14} & & \\ & & x_{8} & x_{12} & x_{16} & & \\ & & x_{8} & x_{12} & x_{16} & & \\ & & x_{6} & x_{7} & x_{10} & x_{16} & & \\ & & x_{6} & x_{7} & x_{10} & x_{16} & & \\ & & & x_{5} & x_{6} & x_{7} & x_{9} & x_{14} & x_{16} \end{vmatrix}$$
(6)

In y_0 , ten arguments, $(x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{14}, x_{16})$ entering into equation (5) twice, therefore, any of them may be taken to perform the expansion. Let us take x_{16} and break the complex event into incompatible events as follows.

$$y_{0} = x_{16} \ y_{00} \lor x_{16} \ y_{01} \ \text{where}$$

$$y_{00} = \begin{vmatrix} x_{1} & x_{5} & x_{9} & & \\ x_{6} & x_{10} & & \\ x_{2} & x_{6} & x_{10} & & \\ x_{5} & x_{9} & & \\ x_{3} & x_{7} & x_{11} & & \\ x_{8} & x_{8} & x_{12} & & \end{vmatrix}$$

$$(7)$$

$$y_{01} = \begin{vmatrix} x_{1} & x_{5} & x_{9} & & \\ x_{6} & x_{10} & & \\ x_{5} & x_{9} & & \\ x_{6} & x_{10} & & \\ x_{5} & x_{9} & & \\ x_{6} & x_{10} & & \\ x_{7} & x_{11} & & \\ x_{8} & x_{12} & & \\ x_{4} & x_{8} & x_{12} & & \\ x_{7} & x_{11} & & \end{vmatrix}$$

$$(8)$$

Now, expand the function y_{00} by argument x_{14} as follows. $y_{00} = x'_{14} y_{000} \lor x_{14} y_{001}$ where

$$y_{000} = \begin{vmatrix} x_1 & x_5 & x_9 \\ x_2 & x_6 & x_{10} \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$
(9)
$$y_{001} = \begin{vmatrix} x_1 & x_5 & x_9 \\ x_2 & x_6 & x_{10} \\ x_2 & x_6 & x_{10} \\ x_5 & x_9 \\ x_3 & x_7 x_{11} \\ x_4 & x_8 x_{12} \end{vmatrix}$$
(10)

Since all the letters occur is equation (9) only once, it implies that y_{000} is non-iterated.

Now, expand the function y_{001} by argument x_9 (say) as follows.

$$y_{001} = x'_{9} y_{0010} \lor x_{9} y_{0011} \text{ where}$$

$$y_{0010} = \begin{vmatrix} x_{1} & x_{5} x_{6} & x_{10} \\ x_{2} & x_{6} x_{10} \\ x_{3} & x_{7} x_{11} \\ x_{4} & x_{8} x_{12} \end{vmatrix}$$

$$y_{0011} = \begin{vmatrix} x_{1} & x_{5} x_{6} & x_{10} \\ x_{2} & x_{6} & x_{10} \\ x_{2} & x_{6} & x_{10} \\ x_{5} & x_{5} \\ x_{3} & x_{7} x_{11} \\ x_{4} & x_{8} x_{12} \end{vmatrix}$$

$$(11)$$

Now expand y_{0010} by argument x_{10} (say) as follows.

$$y_{0010} = x_{10} y_{00100} \lor x_{10} y_{00101} \text{ where}$$

$$y_{00100} = \begin{vmatrix} x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$
(13)

$$y_{00101} = \begin{vmatrix} x_1 & x_5 & x_6 \\ x_2 & x_6 & \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$
(14)

Since all the letters occur is y_{00100} only once, it implies that y_{00100} is non-iterated. Now, expand the function y_{00101} by the argument x_6 (say) as follows.

$$y_{00101} = x'_{6} y_{001010} \lor x_{6} y_{001011} \text{ where}$$

$$y_{001010} = \begin{vmatrix} x_{3} & x_{7} & x_{11} \\ x_{4} & x_{8} & x_{12} \end{vmatrix}$$

$$y_{001011} = \begin{vmatrix} x_{1} & x_{5} \\ x_{2} \\ x_{3} & x_{7} & x_{11} \\ x_{4} & x_{8} & x_{12} \end{vmatrix}$$
(15)
(16)

Since all the letters occur in equation (15) & (16) only once, it implies that y_{001010} and y_{001011} are non-iterated.

Now, we expand the function y_{0011} by argument x_{10} (say) as follows.

$$y_{0011} = x'_{10}y_{00110} \lor x_{10}y_{00111} \text{ where}$$

$$y_{00110} = \begin{vmatrix} x_2 & x_6 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$

$$y_{00111} = \begin{vmatrix} x_1 & x_5 & x_6 \\ x_2 & x_6 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$
(17)
(17)

Since all the letters occur in equation (17) only once, it implies that y_{00110} is non-iterated.

Now, we expand the function y_{00111} by argument x_6 (say) as follows.

$$y_{00111} = x'_{6} y_{001110} \lor x_{6} y_{001111} \text{ where}$$

$$y_{00110} = \begin{vmatrix} x_{3} & x_{7} & x_{11} \\ x_{4} & x_{8} & x_{12} \end{vmatrix} \text{ which is non-iterated}$$
(19)

$$y_{001111} = \begin{vmatrix} x_1 & x_5 \\ x_2 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{vmatrix}$$
(20)

Since all the letters occur in equation (19) only once, it implies that y_{001110} is non-iterated.

Proceeding in this way, finally it is observed that all the functions are non-iterated and also not subjected to further transformations.

So making use of equations, we get

 $g(x_1 x_2 \dots x_{12}, x_{14}, x_{15}, x_{16}) =$

18	X ₁₀	$ \vec{x}_{14} $	X _I	X3	Xg				
1			X7	Xe	Xm				
			X;	X7	Xm				
			\mathbf{x}_{t}	X,	X_{12}				
		χ_{pq}	X _p	1 X ₁₀	X	X 7	\mathbf{x}_{H}		
					\mathbf{x}_{t}	X ₂	X_{12}		
				X10	×.	X ₂	X7	\mathbf{x}_{II}	
						\mathbf{x}_{t}	χ_{δ}	X_{12}	
					Xs	X _I	X ₂		
						X_2			
						Xi	X_7	X_{II}	
						\mathbf{x}_{t}	X_{δ}	X_{II}	
			χ_{p}	×10	X2	χ_{3}	X_{δ}		
					X_{2}	\mathbf{x}_7	\mathbf{X}_{II}		
					\mathbf{x}_{t}	Xa	\mathbf{X}_{II}		
				X10	X _o	X;	X 7	X_{II}	
						xe	X ₂	X_{II}	
					Xe	x _s	X ₂	X 7	x_{II}
							\mathbf{x}_{t}	X ₂	χ_{D}
						X3	X,		
							\mathbf{X}_2		
							X	X_7	X_{D}
					1	1	1 x.	¥	10 m

Х ₁₅	X16	\dot{X}_{s}	X,	x2	χ_{δ}	X_{I0}				
				\mathbf{x}_{t}	X	X_{D}				
			X7	X ₁₁	X	X2	Х <u>а</u>	X_{10}		
					X ₄	X ₁₂	X2	X_{δ}	X_{10}	
						X_{II}	X2 ×	Xø	X10	
							20 20			
				X ₁₁	X.	X2	Xe	X10		
					χ_{4}	$\dot{\mathbf{x}}_{12}$	X2	Xs	X10	
						χ_{D2}	X ₂	χ_{δ}	χ_{10}	
							Xi			
							X _t			
		X ₂	×.	×.	\mathbf{x}_{l}	x ₂				
			D		X,	X ₂	X12			
						I I				
				X 7	X	Xj	Xg			
						Xi	\mathbf{x}_{II}			
					X ₀	\vec{x}_{0}	$ X_{12}$	X,	X ₂	
							X12	Xr	Xş	
								X; Y.		
						X_{II}	\mathbf{x}_{12}	X ₂	Xş	
							¥	v .	x .	
							412	X;	A.	
								\mathbf{x}_{t}		
				<u>.</u>	l					
			Xe		×9	×10	<i>X</i> ₄	X.	X12	
						A10	Xe Xe	114		
							\mathbf{x}_{t}	$\chi_{\rm g}$	X_{D2}	
						Ι.				
					Xg	\mathbf{x}_{14}	X ₂	X_{10}		
							X _f	$\mathbf{x}_{\mathbf{s}}$	X_{I2}	
						XH	$\dot{X}_{\rm m}$	\mathbf{x}_{t}	Xa	X12
							X10	X ₁		
								X2		
	•	•	•	•	•	•	•	χ_{ℓ}	X_{δ}	X_{JZ}

						r-	<u>.</u>	r	T -	
						~	A.	X2 X2 X2	x ₁₀ x ₁₁	X _{Id}
	_	_					Xs	X1 X2 X1 X4	X9 X10 X11 X12	XII
X13	X_{16}	X_{14}	X ₁₁	X_{10}	X_I	χ_{j}	Xg	-	-	
					\mathbf{x}_{t}	χ_{δ}	X_{II}			
				X10	X _e	X	X3	Xe		
						X _t	$\chi_{\rm g}$	χ_{12}		
					Xe	X) X2	X3	Xg		
						X	X 7			
						X _f	χ_{d}	χ_{12}		
			X _{II}	x ₁₀	X ₂	X ₂	Xe			
					X2	χ_{δ}	X 7			
					\mathbf{x}_{t}	χ_{g}	χ_{D}			
				X10	×,	X	X3	Xe		
						Xe	χ_{s}	X_{12}		
					X7	×.	1 . X1	X.	X.	
						D	\mathbf{x}_{t}	X ₀	X12	
						¥.,	v .	v .	v .	
						-46	X2	- 43	2.9	
							X			
							\mathbf{x}_{t}	χ_{δ}	χ_{12}	
		\mathbf{X}_{M}	X	X_{p}	Х ₁₀	X _e	χ_{s}	χ_{12}		
					X10	X _a	x,	Xa	X_{12}	
						Xe	X,	X3		
							X_2			
			Υ.			1	X;	x7		
			~	~~ X2	X	x	$\int X_{\ell}$	Xa	X_{12}	
					10	2	1	1		

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1			1	I I	I I	I I	l	1			
							-16	A2 - X	*		
								A.; 	X7 X0	Y	
					×	- V	v .	·	- 44 X - 4	412	
					A10	1	2	- 44 - 2	A10		
						18	×.	14	14	X10	
							X3	X) X			
								A2 7-	Y-		
								1	X-	Xm	
			X11	×	×.	×.	X,	X ₂	X12		
				10	, °	X7	x	x.	Xa	X12	
							Xe	x	Xe		
								X2			
								Xe	Xa	X_{12}	
						Ι.					
					Xç	X,	X2	Xs	X3		
							X _e	Xa	X_{12}		
						X7	X ₆	X2	Xs	X3	
								X _e	Xa	X12	
							Xs	X _s	xe	Xa	X12
								X3	Xr		
									X2		
									Xz		
									Xe	Xa	X12
				X10	×,	×, .	X _I	X3	Xs		
							X _e	Xa	X12		
						X7	X _e	Xe	Xa	X12	
							Xs	X _J	X3		
								Xz	-		
								X			
								X _e	Xa	X_{12}	
					Xe	×	×.	x	x,	X.,	
					- T		x	×	L x	x	X12
							~	x.	x.	-	
								~	x		
							I		\mathbf{x}_{t}	X_{δ}	X12
X16	$\dot{\mathbf{x}}_{14}$	X ₁₂	x ₁₁	X ₁₀	X ₂	X3	Xp				
				X10	$\dot{\mathbf{x}}_{\mathrm{s}}$	Xr	X3	Xg			
					Xe	x,	\mathbf{x}_{t}	X3	Xg		
I							×.				

X_{16} X_{14} X_{17} X_{17} X_{10} X_{10} X_{1} X_{10} X_{1} X_{10} X_{1} X_{1} X_{2} X_{2						ĺ	X 7	$\begin{array}{c} \chi_{1} \\ \chi_{2} \end{array}$	X3	Xg	
$\mathbf{X}_{16} = \begin{array}{ccccccccccccccccccccccccccccccccccc$								X			
X_{12} X_{12} X_{12} X_{12} X_{13} X_{13} X_{13} X_{14} X_{15} X_{15} X_{16} X_{15} X_{16} X_{1} X_{1} X_{2} $X_{2} X_{2} X$							χ_t	X_{δ}			
X_{12} X_{13} X_{13} X_{12} X_{13} X_{12} X_{13} X_{13} X_{14} X_{15} X				X _{II}	X ₁₀	X,	X_{I}	X_2	Xg		
X_{126} X_{14} X_{14} X_{12} X_{13} X_{14} X_{15} X_{15} X_{15} X_{15} X_{16} X_{17} X_{18} X_{19}							X2	Xe			
X_{13} X_{14} X_{12} X_{11} X_{11} X_{11} X_{10} X_{1} X_{2} X_{3} X_{4} X_{4} X_{4} X_{4} X_{4} X_{5}					Yes	x'	Xe Xe	ла У.	Y.		
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$egin{array}{c c c c c c c c c c c c c c c c c c c $							\mathbf{x}_{t}	X_{δ}			
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X_{10} X_{10} X_{1} X_{2} X_{3} $X_$						Xa	X ₇	X_I	X3	χ_{g}	
X_{10} X_{1} X_{2} X_{3} X_{4} X_{2} X_{3} X_{4} X_{10} X_{1} X_{2} X_{3} X_{4} X_{2} X_{3} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{12} X_{2} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{12} X_{2} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{11} X_{1} X_{2} X_{3} X_{4} X_{11} X_{12} X_{2} X_{3} X_{4} X_{12} X_{1} X_{2} X_{3} X_{4} X_{2} X_{3} X_{3} X_{4} X_{2} X_{3} X_{4} X_{3} X_{4} X_{3} X_{5} X_{4} X_{4} $X_{$							X7	X_{I}	χ_{j}	Xg	
$egin{array}{ c c c c c c c c c c c c c c c c c c c$								л. Х:	18		
$\left \begin{array}{c c c c c c c c c c c c c c c c c c c $											
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$egin{array}{ c c c c c c c c c c c c c c c c c c c$							Xa	X_7	X_{δ}		
$egin{array}{ c c c c c c c c c c c c c c c c c c c$								I			
$egin{array}{ c c c c c c c c c c c c c c c c c c c$						X	×,	<i>x</i> ₁	X3	Xg	
$egin{array}{c c c c c c c c c c c c c c c c c c c $							X7	X ₆	Xj	X3	Xg
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$\left \begin{array}{cccccccccccccccccccccccccccccccccccc$							•	•	24 Xe		
$egin{array}{c c c c c c c c c c c c c c c c c c c $				X _{II}	×,	X _I	X ₅	χ_{g}			
$\left \begin{array}{c cccccccccccccccccccccccccccccccccc$						X ₂	$\chi_{\rm S}$	<i>X10</i>			
$egin{array}{c c c c c c c c c c c c c c c c c c c $					X7	Х ₁₀	X	X,	X3	Xe	
$\begin{array}{ $							Xa	х, х	X3	Xg	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$.A.2 Y.	15		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$						X10	×.	X ₁	X ₂	Xo	
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X6 X1 X2 X2 X1 X2 X2 X2 Y2 Y2 Y2 Y2							Xs	X ₆	\mathbf{x}_{i}	X_3	Xg
								Xe	X _J	χ_{g}	χ_{g}
Y.									Х2 Х-		
-									Υ.		

x_{Id}	\mathbf{X}_{S}	X ₆	X,	\mathbf{x}_{t}	Xa	X_{12}			
			X7	X,	Xi	\mathbf{X}_{11}			
				X_{2}	X3	X_{IJ}			
					\mathbf{x}_{t}	χ_{I2}			
		Xs	×.	x2	X10				
		_	r	Xe	XJ	χ_{12}			
			Y-	, x	v .	Yes			
			~	15	\sim	~			
				×.		All Yes			
				~	x	Xo			
					X.	X ₁₂			
	Xs	X _e	X ₁₂	× _Π	Xr	Xg			
				X_{II}	X,	\mathbf{x}_{l}	Xp		
					X7	\mathbf{X}_{I}	χ_{p}		
						X_{2}			
						\mathbf{x}_{t}	Xa		
			X12	×.	X,	Xg			
					X ₂	X7	X_{II}		
				X ₂	x.	x,	X ₂		
				_	\mathbf{x}_{D}	×.	X,	χ_p	
						X7	X,	Xe	
							X		
					1		\mathbf{x}_{t}		
		χ_{δ}	X,	X_I	χ_{g}		•		
				X_2	X ₁₀				
				X _t	X	χ_{12}			
			X7	X	Xg				
				X ₂	X_{10}				
				X;	χ_{g}	X_{12}			

Now using Baye's formula, the probability of successful operation of the function g is given by

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$$\Pr(g=1) = \sum_{i=1}^{94} \Pr(K_i) \Pr(g / K_i)$$
(22)

If R_i is the reliability of the component of the complex system corresponding to state x_i and Q_i is the corresponding unreliability, then from equation (22) finally, the probability of the successful operation (i.e., reliability) of the complex system is given by

$$R_s = Pr (f = 1) = Pr(x_{13}).Pr (g = 1)$$

Particular Cases

If reliability of each component of the complex system is R, then $R_s = 4R^4 - 12R^5 + 17R^6 - 20R^7 + 16R^8 + 10R^9 - 34R^{10} + 36R^{11} - 17R^{12} - 3R^{13} + 10R^{14} - 8R^{15} + 2R^{16}$.

When failure rates follow Weibull distribution, let failure rate of each component of the complex system be λ , then reliability of the system at an instant t is given by

$$R_{s}(t) = 4e^{-4\lambda t^{\alpha}} - 12e^{-5\lambda t^{\alpha}} + 17e^{-6\lambda t^{\alpha}} - 20e^{-7\lambda t^{\alpha}} + 16e^{-8\lambda t^{\alpha}} + 10e^{-9\lambda t^{\alpha}} - 34e^{-10\lambda t^{\alpha}} + 36e^{-11\lambda t^{\alpha}} - 17e^{-12\lambda t^{\alpha}} - 3e^{-13\lambda t^{\alpha}} + 10e^{-14\lambda t^{\alpha}} - 8e^{-15\lambda t^{\alpha}} + 2e^{-16\lambda t^{\alpha}}$$

where α is a positive parameter.

When failure rates follow Exponential distribution, exponential distribution is a particular case of Weibull distribution for $\alpha = 1$. The reliability of the complex system in this case at an instant t is

$$R_{s}(t) = 4e^{-4\lambda t} - 12e^{-5\lambda t} + 17e^{-6\lambda t} - 20e^{-7\lambda t} + 16e^{-8\lambda t} + 10e^{-9\lambda t} - 34e^{-10\lambda t} + 36e^{-11\lambda t} - 17e^{-12\lambda t} - 3e^{-13\lambda t} + 10e^{-14\lambda t} - 8e^{-15\lambda t} + 2e^{-16\lambda t}$$



 $\lambda = 0.1, \alpha = 2.$

Conclusion

This graph computes the variation of reliability with respect to time, when failure follows Exponential and Weibull distributions. A critical examination of Reliability Vs Time graph shows that the reliability of the complex system decreases approximately at a uniform rate in case of Exponential distribution, whereas it decreases very rapidly when failure follows Weibull distribution.

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