Advances in Applied Mathematical Biosciences.
ISSN 2248-9983 Volume 5, Number 2 (2014), pp. 103-113

http://www.irphouse.com

Mathematical Modeling of Unsteady Flow Through A Tube With Time Dependent Stenosis in Hemodynamics

Rashid Ali[a], Monika Gupta[b], M.P.Singh[b] and V.K.katiyar[c]

[a]Department of Mathematics, Krishna Engineering College, Ghaziabad, U.P., India
raliarsh@rediffmail.com
[b]Department of Mathematics, Gurukul Kangri University Haridwar, Uttrakhand, India,
meeshu.raman@gmail.com
[c]Department of Mathematics, IITR Roorkee, Uttrakhand, India

Abstract

In this paper, a mathematical model is developed, taking into consideration the slip velocity at the wall of blood vessel of time-dependent Stenosis with unsteady flow through a tube in the presence of a Stenosis. Momentum integral method has been used to evaluate axial velocity and pressure gradient in time dependent Stenosis. Results have also been compared with and without the effect of slip velocity. It has been observed that axial velocity differ significantly with existence of slip velocity. The present study asserts that the slip velocity has a reducing effect on the pressure drop.

Keywords : Unsteady Flow, Integral Momentum Method, slip velocity, stenosis.

Introduction

Fat and cholesterol deposits on the inner walls of arteries cause narrowing of the lumen diameter, resulting in a decrease in blood flow to the region upstream of the constriction. Depending on the artery affected, the reduction in flow may result in stroke, gangrene, angina, or even a heart attack. Certain hydro dynamical factors play a very important role in the development of this disease. Lee and Fung [1] employed numerical technique to solve a problem concerning blood flow through a stenosed tube. Haldar [2] had obtained analytical results for oscillatory flow of blood which behaves as a Newtonian fluid.

McDonald [3] remarked that for vessels of radius greater than 0.25 mm., blood may be considered as a homogenous Newtonian fluid. At low shear rates blood exhibits non-Newtonian behaviour [4], in larger arteries where the shear rate is considerably high, blood may be represented as a Newtonian fluid. Bennett [5] who carried out in vitro experiments to study that behaviour of red cells during blood flow, suggested that there might exist the possibility of red cells to have a slip velocity at the wall under certain conditions. Subsequently several other investigators e.g. Nubar [6], Brunn [7] and Mishra and Kar [8] also indicated the possibility of a slip-velocity at the inner surface of the vessel wall. Cavalcanti [13] discussed hemodynamics of an artery with mild Stenosis. Liu and Yamaguchii [14] studied waveform dependence of pulsatile flow in a stenosed channel Monika and Bortila [15] presented the simulation of incompressible non- Newtonian flow through channels with sudden expansion using the power law model. Sankar and Hemalatha [16] have studied a non-Newtonian fluid flow model for blood flow through a catheterized artery-steady flow. Massoudi and Phuoc [17] considered pulsatlile flow of blood using a modified second-grade fluid model. Ismail et al. [18] discussed a power-law model of blood flow through a tapered overlapping stenosed artery. Chakraborti [19] found that the pulsatile blood flow through an inclined stenosed tube with a velocity slip Riahi et al. [20] considered unsteady blood flow in an artery with an overlapping stenosis. Kumar and Diwakar [21] discussed A mathematical model for Newtonian blood flow in the presence of applied magnetic field. Recently, Ali et al. [22] have studied the mathematical modelling for the flow of blood cells in capillaries. Mohan et al. [23] considered the effect of magnetic field on blood flow (elastico-viscous) under periodic body acceleration in porous medium.

In the present paper, a mathematical model has been developed and it has been shown that pressure drop in constricted tube increases as time increases but it reduces as slip-velocity increases.

Stenosis Model

By Mishra and kar [8], it is obvious that Stenosis (constriction) has no well defined geometrical configuration. In general, complex three-dimensional flow patterns have been developed near the stenosis which is virtually impossible to analyse. In this paper, 'collar like' stenosis model i.e. axisymmetric constriction in a long tube has been considered. It is true that the size of stenosis increases with time and attains some fixed geometrical configuration after some time. It is assumed that flow is unsteady and laminar, the artery is of constant diameter ($2R_o$) preceding and following the stenosis.

Fig. 1 describes the stenosis geometry in the cylindrical polar coordinate system. It is further assumed that stenosis grows in an axially symmetric manner due to abnormal growth over a length $2L_o$ of the artery. Tondon and Katiyar [11] expressed the local radius (R) of the axisymmetric tube as a function of longitudinal coordinate x and time t as

$$R = R_o - \frac{\delta}{2}\left(1 - e^{\frac{-t}{T}}\right)\left(1 + \cos\frac{\pi x}{L_o}\right), \tag{1}$$

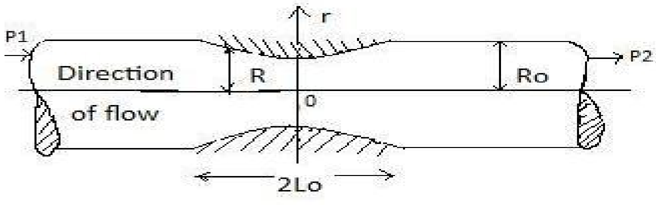

Fig. 1 Geometry Of Stenosis

where $R = R_o$ for $t = 0$, δ is constant (height of stenosis). $\frac{t}{T}$ is time function and T is time constant for stenotic growth. However, it should be noted that although the growth rate as characterized by the parameter, T, is not important to the fluid mechanics of the problem, the rates at which variables such as pressure and shearing stress are changing may play an important role in certain cellular process.

Analysis

For two-dimensional flow, the governing equations for the flow of a Newtonian, viscous, incompressible fluid may be taken in the form in axial direction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right) \tag{2}$$

and in the radial direction

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial r^2}-\frac{v}{r^2}\right) \tag{3}$$

while the continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0, \tag{4}$$

where *u* represents the axial velocity, v the radial velocity, ρ the density, *p* the pressure and ν the kinematic viscosity coefficient of the blood.

During the initial development of the stenosis $\frac{\delta}{R_0} << 1$ and $v << u$ then radial variation of pressure i.e. $\frac{\partial p}{\partial r}$ may be neglected and $\frac{\delta}{L_0} << 1$ then normal stress gradient $\frac{\partial^2 u}{\partial x^2}$ is negligible compared to the stress gradient $\frac{\partial^2 u}{\partial r^2}$.

The governing equations can be approximated for mild stenosis as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{5}$$

and $$\frac{\partial p}{\partial r}=0. \tag{6}$$

By applying the integral momentum method and using the continuity equation, integrating equation (5) over the cross-section of vessel, we obtain the integral equation as

$$\frac{\partial}{\partial t}\int_0^R ru\,dr+\frac{\partial}{\partial x}\int_0^R ru^2dr=-\frac{1}{\rho}\left(\frac{R^2}{2}\right)\frac{\partial p}{\partial x}+\nu R\left(\frac{\partial u}{\partial r}\right)_R . \tag{7}$$

At the surface, the boundary condition is $v=0$ *and* $u=W$ (slip velocity) at $r = R$.

The volume flux is given by

$$Q=2\pi\int_0^R ru\,dr , \tag{8}$$

we choose the velocity profile in the form of as

$$\hat{u}=\frac{u}{U}=A+B\eta+C\eta^2+D\eta^3+E\eta^4, \tag{9}$$

where $\eta=\left(\frac{R-r}{R}\right)$ (9a)

U being the centre line velocity and A,B,C,D and E are constants to be determined from the velocity constraints. Using (9) and (9a), equation (8) may be rewritten as

$$Q=2\pi R^2 U\int_0^1(1-\eta)\hat{u}\,d\eta. \tag{9b}$$

The appropriate boundary conditions are

$u=U \quad at \quad r=0,$ (10a)

$u=W \quad at \quad r=R,$ (10b)

(10c)

s

$\frac{\partial^2 u}{\partial r^2}=-2\frac{U}{R^2} \quad at \quad r=0,$ (10d)

together with

$$\frac{\partial p}{\partial x}=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad at \quad r=R. \tag{10e}$$

The first one describes the centre line velocity, second is the slip condition at the wall, third describes the viscous stress which is proportional to $\frac{\partial u}{\partial r}$ and which should approach zero as the radius of the tube approaches zero when other forces are finite.

The second radial derivative of u at $r=0$ may be approximated by the fourth condition assuming the velocity profile to be nearly parabolic at the axis as represented by the Poiseuille;s profile $\hat{u}=1-(r/\ R)^2$. Finally, the fifth condition represents the validity of equation (5) at $r=R$. The above written condition transformed in terms of variable η introduced in (9a) may be put as

$\hat{u}=1 \quad at \quad \eta=1,$ (11a)

$\hat{u}=\frac{W}{U} \quad at \quad \eta=1,$ (11b)

$\frac{\partial \hat{u}}{\partial \eta}=0 \quad at \quad \eta=1,$ (11c)

$$\frac{\partial^2 \hat{u}}{\partial \eta^2} = -2 \qquad at \quad \eta = 1, \tag{11d}$$

and

$$\frac{\partial p}{\partial x} = \frac{\mu U}{R^2}(1-\eta)\frac{\partial}{\partial \eta}\left[(1-\eta)\frac{\partial \hat{u}}{\partial \eta}\right] \quad at \quad \eta = 0, \tag{11e}$$

Using conditions (11a) to (11e), the velocity profile is as follows

$$\begin{aligned}\hat{u} = \frac{u}{U} = \left(\frac{-\lambda+10-12g}{7}\right)\eta + \left(\frac{3\lambda+5-6g}{7}\right)\eta^2 + \left(\frac{-3\lambda-12+20g}{7}\right)\eta^3 \\ + \left(\frac{\lambda+4-9g}{7}\right)\eta^4 + g \ ,\end{aligned} \tag{12}$$

where

$$\lambda = \frac{R^2}{\mu U}\frac{d\,p}{d\,x} \quad and \quad g = \frac{W}{U}. \tag{13}$$

Here the velocity profile becomes a function of only one parameter λ, which is function of pressure gradient $\frac{\partial p}{\partial x}$.

Substituting (12) into (9b) and taking the integration we get

$$U = \frac{210}{97\pi R^2}\left[Q + \frac{\pi R^2}{105\mu}\frac{d\,p}{d\,x} + \frac{78\pi R^2 W}{210}\right] \tag{14}$$

The parameter λ may be determined from equation (7) as

$$\lambda = \frac{4}{5}(6g-5) - \frac{14}{5\nu U}\frac{\partial}{\partial t}\left[R^2 U\int_0^1 (1-\eta)u\,d\eta\right] - \frac{14}{5\nu U}\frac{\partial}{\partial x}\left[R^2 U^2\int_0^1 (1-\eta)u^2 d\,\eta\right], \tag{15}$$

In equation (15), neglecting the terms in velocity profile, higher than two because profile according to Bugliarello [9] due to specific diameter $(5.0\,\mu\,m)$ of the arteries is as

$$u = 2\overline{U}\left[1-(r/R)^2\right], \tag{16}$$

where

$$\overline{U} = -\left(R^2/8\mu\right)\frac{d\,p}{d\,x}\ , \tag{17}$$

$\overline{U}$ is the average velocity of cross-section and $\frac{d\,p}{d\,x} < 0$.

Substitution of (16) into momentum integral equation (7) yields

$$\frac{\partial}{\partial t}\left(\frac{\overline{U}\,R^2}{2}\right) + \frac{\partial}{\partial x}\left(\frac{2}{3}\overline{U}^2 R^2\right) = -\frac{1}{\rho}\left(\frac{R^2}{2}\right)\frac{d\,p}{d\,x} + \frac{\nu \overline{U}}{7}(\lambda - 10 + 12g). \tag{18}$$

Using $\overline{U} = Q/\pi R^2$ and combining the resulting equation with equation (14). The pressure gradient is obtained in the form

$$\frac{d\,p}{d\,x} = 1.5\frac{\mu W}{R^2} - 8\frac{\mu \overline{U}}{R^2} - 2.9\frac{\overline{U}}{R}\frac{\partial R}{\partial t} - 3.7\frac{\overline{U}^2}{R}\frac{\partial R}{\partial x}. \tag{19}$$

The first term on the right hand side of equation (19) is due to the slip velocity, the second term is due to the viscous shearing stress, third due to rate of change with time and fourth term is due to inertia of blood.

In non-dimensional form, equation (19) read as

$$\left(\frac{R_0}{U_0\rho}\right)\frac{dp}{dx}=\frac{3}{\mathrm{Re}_0}\left(\frac{R_0}{R}\right)^2\frac{W}{U_0}-\frac{16}{\mathrm{Re}_0}\left(\frac{R_0}{R}\right)^2-2.9\left(\frac{R_0}{R}\right)\frac{1}{U_0}\frac{\partial R}{\partial t}-3.7\left(\frac{R_0}{R}\right)\frac{\partial R}{\partial x}\ , \qquad (20)$$

Where $\mathrm{Re}_0=\left(\dfrac{2R_0\overline{U}_0\rho}{\mu}\right)$ (21)

is the Reynolds number upstream from the stenosis, $\overline{U}_0$ being the average velocity.

Substituting (14) and (19) into (12), we obtain the velocity distribution u as a function of r and x as follows

$$\begin{aligned}\hat{u}=\frac{u}{\overline{U}_0}&=\left(1.4\eta-5.6\eta^2+5.9\eta^3-2\eta^4\right)\frac{\mathrm{Re}_0}{7}\left(\frac{R}{R_0}\right)\frac{\partial R}{\partial x}\\&+\left(1.2\eta-4.7\eta^2+4.9\eta^3-1.6\eta^4\right)\frac{\mathrm{Re}_0}{7}\left(\frac{R}{R_0}\right)\frac{\partial R}{\partial t}\\&+\frac{W}{7\overline{U}_0}\left(7-5.5\eta+2.5\eta^2+5.5\eta^3-4.5\eta^4\right)\qquad (22)\\&+\left(22\eta+11\eta^2-26.4\eta^3+8.8\eta^4\right)\frac{1}{7}\left(\frac{R_0}{R}\right)^2\\&+\frac{1}{7}\left(6\eta-2.5\eta^2+26.4\eta^3-8.8\eta^4\right)\end{aligned}$$

The skin-friction τ_w is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R} \qquad (23)$$

Employing (22), we further derive the following expression for the skin-friction

$$\frac{\tau_w}{\rho\overline{U}_0^2}=\frac{60}{7}\left(\frac{R_0}{R}\right)\frac{1}{\mathrm{Re}_0}\frac{W}{\overline{U}_0}-0.4\frac{\partial R}{\partial x}-\frac{2.4}{7}\frac{\partial R}{\partial t}-\frac{44}{7}\frac{1}{\mathrm{Re}_0}\left(\frac{R_0}{R}\right)^3-\frac{12}{7}\frac{1}{\mathrm{Re}_0}\left(\frac{R_0}{R}\right).(24)$$

Results and Discussion

The slip velocity has been taken to be equal to 10% the average velocity of the blood in a normal artery. The calculation has been carried out at the locations defined by $\frac{x}{L_0}=0,\pm 0.5$ and ± 1.0 for three different values of Reynolds number, viz. $\mathrm{Re}_0=100$, 300 and 500 for different time parameters $\frac{t}{T}=0$ and 3.

Figs. 2 and 3 depict the variation of the non-dimensional axial velocity of blood flow in the stenosed arterial segment with and without consideration of slip velocity. The variation of the non-dimensional axial velocity of blood flow in the stenosed arterial segment with and without consideration of the slip velocity has been depicted through figs. 2, 3, 4, 5 and 6, 7 for $\mathrm{Re}_0=100$, 300 and 500 respectively for different values of time parameter. It may be observed that with the increase in

Reynolds number, the axial velocity in the presence of slip velocity at the well appreciably differ from the corresponding velocity in the absence of slip velocity. The difference between the velocity with slip velocity and velocity without slip is decreasing. Hence axial velocities are also increasing with time and Reynolds numbers at $\frac{x}{L_0} = 0, \pm 0.5$ and ± 1.0.

Conclusion

We shall use the following geometrical configuration of the stenosis, used b Young [10] and Tondon and Katiyar [11] as

$$R = R_0 - \frac{\delta}{2}\left(1 - e^{-t/T}\right)\left(1 + \cos\frac{\pi x}{L_0}\right). \tag{26}$$

The stenosis parameter $R_0,\ L_0,\ and\ \delta$ are taken to be related as

$$L_0 = 4R_0 = 12\delta. \tag{27}$$

The pressure drop in its non-dimensional form may be obtained by integrating equation (20) from the upstream end of stenosis to any point along it, is given by

$$\frac{p_0 - p}{\rho \bar{U}_0^2} = \frac{1}{R_0}\left[\frac{16\bar{U}_0}{\mathrm{Re}_0}\int_{-L_0}^{x}\left(\frac{R_0}{R}\right)^2 dx + \frac{2.9}{\bar{U}_0}\int_{-L_0}^{x}\left(\frac{R_0}{R}\right)\frac{\partial R}{\partial t}dx + 3.7\int_{-L_0}^{x}\left(\frac{R_0}{R}\right)dx - \frac{16.6}{\mathrm{Re}_0}\frac{W}{\bar{U}_0}\int_{-L_0}^{x}\left(\frac{R_0}{R}\right)^2 dx\right], \tag{28}$$

where p_0 represents the pressure at the upstream end $x = -L_0$ of the stenosis. Table – 1 gives the values of the non-dimensional pressure drop for $\mathrm{Re}_0 = 2\bar{U}_0 R_0/\nu = 500$ at different locations of the stenosed artery for four different values of the parameter for different values of $\frac{t}{T} = 0, 1, 2,$ and 3. T The values obtained by Forrester and Young [8] have also been shown for comparison on idea of the extent to which the slip velocity affects the pressure drop.

Dimensionless pressure drops between two axial positions one upstream and another downstream of stenosis are obtained by integrating equation (20) from $x = -4L_0$ to $x = 4L_0$

$$\frac{\Delta p}{\rho \bar{U}_0^2} = \left[\frac{32\bar{U}_0}{R_0\,\mathrm{Re}_0} - \frac{33.2}{R_0\,\mathrm{Re}_0}\frac{W}{\bar{U}_0}\right]\int_0^{4L}\left(\frac{R_0}{R}\right)^2 dx, \tag{29}$$

where $\left(\frac{R_0}{R}\right) = \frac{6}{\left[5 + e^{-t/T} - \left(1 - e^{-t/T}\right)\cos\frac{\pi x}{4L_0}\right]}.$

For a straight tube, $\left(\frac{R_0}{R}\right) = 1$, so that equation (20) reduces to

$$\frac{R_0}{\rho \bar{U}_0^2}\frac{dp}{dx} = \left[16.6\frac{W}{\mathrm{Re}_0\,\bar{U}_0} - \frac{16}{\mathrm{Re}_0}\right]. \tag{30}$$

Integrating equation (30) between $x = -4L_0$ to $x = 4L_0$, the pressure drop in unobstructed artery is

$$\frac{\Delta p}{\rho \bar{U}_0^2} = \frac{1}{\mathrm{Re}_0}\left[128 - 132.8\frac{W}{\bar{U}_0}\right]. \tag{31}$$

The pressure drops have been calculated by the expressions (29) and (31) for stenosed and normal arteries using the data. These results have been shown in Fig. 8 along with Forrester and Young [12] for the purpose of comparison.

Fig. 8 depicts the variation of pressure drop with Reynolds numbers in the presence of stenosis. The velocities get elevated in comparison to Mishra and Kar's model [8] of stenosis for unobstructed artery. These observations are in agreement with experimental observations of Forrester and young [12]. Hence slip velocity has a reducing effect on the pressure drop.

TABLE – 1 Distributions of the non-dimensional pressure drop with time and slip velocity.

	Time $\frac{t}{T}$	$\frac{W}{\bar{U}_0}$	X/L_0			
			$-1/2$	0	$1/2$	1
With consideration of slip velocity (Present investigation)	0	0.1	0.694	0.694	0.694	0.694
		0.25	0.692	0.692	0.692	0.692
		0.50	0.688	0.688	0.688	0.688
	1	0.1	0.866	1.114	0.866	0.694
		0.25	0.863	1.110	0.863	0.692
		0.50	0.858	1.103	0.858	0.688
	2	0.1	0.948	1.369	0.948	0.694
		0.25	0.944	1.364	0.944	0.692
		0.50	0.939	1.356	0.936	0.688
	3	0.1	0.980	1.486	0.980	0.694
		0.25	0.976	1.482	0.976	0.692
		0.50	0.971	1.472	0.971	0.688
Disregarding slip velocity (Forrester and Young [8])	0	0	1.5	5.2	3.8	3.2

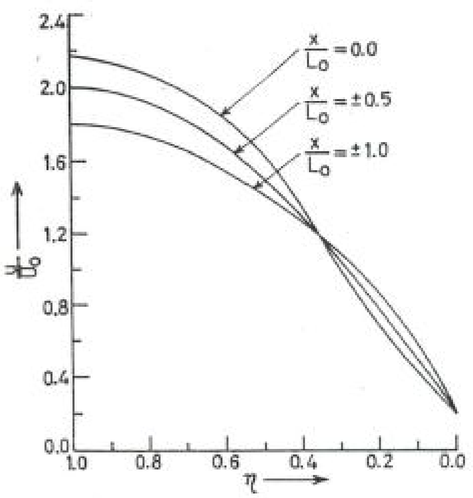

Fig. 2. Variation of axial velocity for $t/T = 0$, $R_{eo} = 100$ with slip velocity.

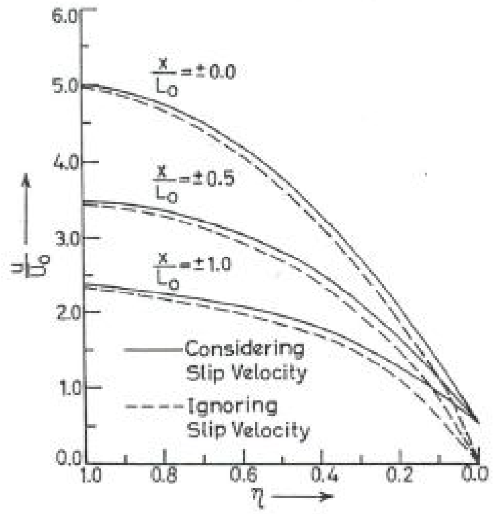

Fig. 3. Variation of axial velocity for $t/T = 3$, $R_{eo} = 100$ with & without slip velocity.

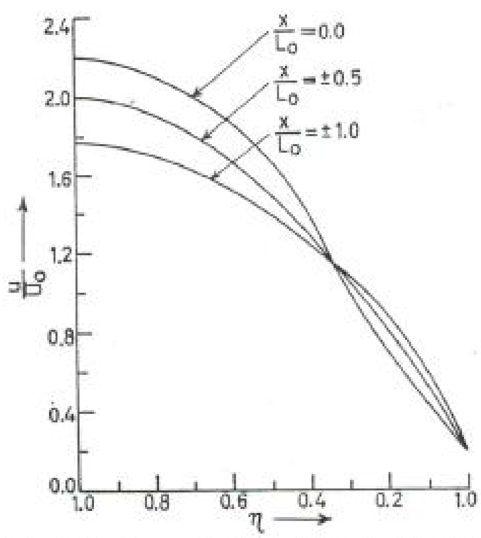

Fig. 4. Variation of axial velocity for $t/T = 0$, $R_{eo} = 300$ with slip velocity.

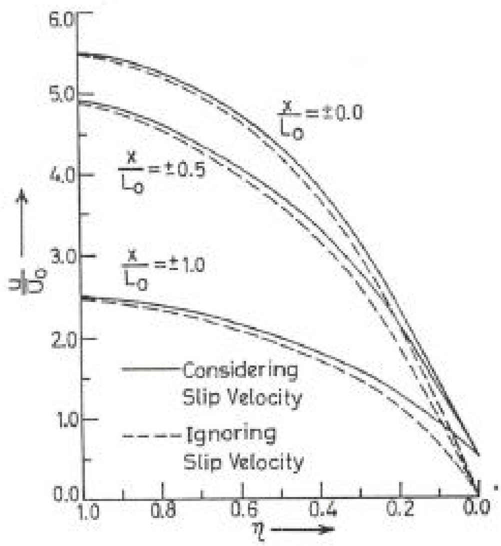

Fig. 5. Variation of axial velocity for $t/T = 3$, $R_{eo} = 300$ with & without slip velocity.

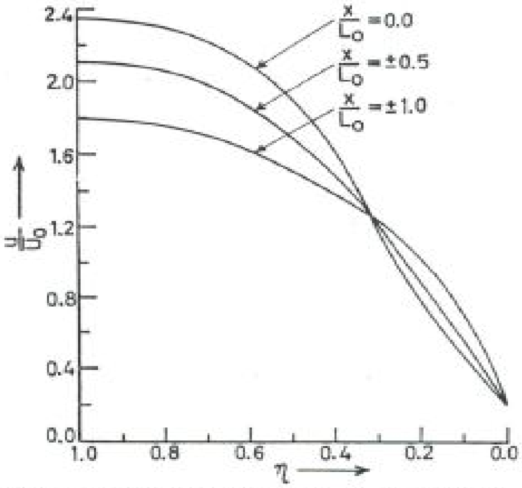

Fig.6. Variation of axial velocity for t/T=0, R_{eo}=500 with slip velocity.

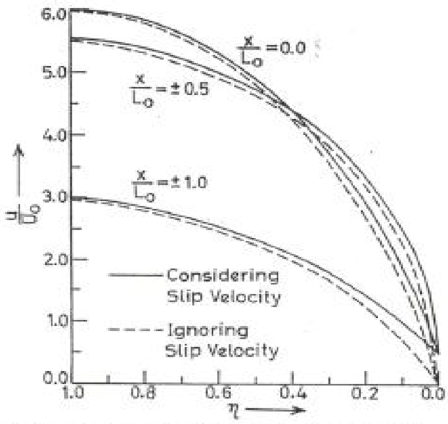

Fig.7. Variation of axial velocity for t/T=3, R_{eo}=500 with & without slip velocity.

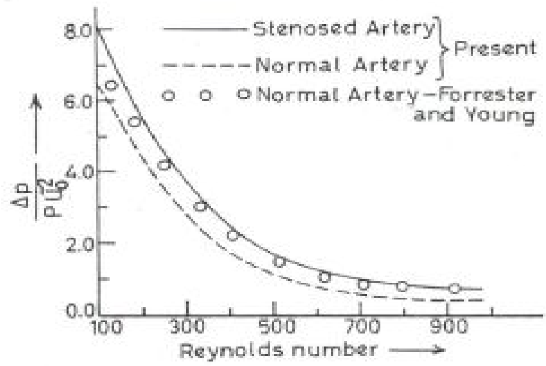

Fig.8. Variation of non-dimensional pressure drop with Reynolmds numbers.

References

[1] Lee, J.S. and Fung, Y.C., 1970, Flow in locally constricted tubes at low Reynolds number, J. Appl. Mech., 37, 9-16.
[2] Haldar, R., 1987, Oscillatory flow of blood in stenosed artery, Bulletin of Mathematical Biology, 49:3, 279-287.
[3] Mcdonald, D.A., 1974, Blood flow in arteries, Edward Arnold Ltd., 133.
[4] Merrill, E.W., 1965, Rheology of human blood and some speculation on its role in vascular homeostatic and intravascular thrombosis (Ed. P.N. sawyer), Appleton Century Crafts 121-137.
[5] Bennett, L., 1967, Red cell slip at wall in vitro, Science, 15, 1554-1556.
[6] Nubar, Y., 1971, Blood flow, slip and viscometry, Biophysics, J., 11, 252-264.

[7] Brunn, U., 1975, The velocity slip of polar fluids, Rheol. Acts. 14, 1039-1054.

[8] Mishra, J.C. and Kar, B.K., 1989, Momentum integral method for studying flow characteristics of blood through a stenosed vessel, biorheology, 26, 23-35.

[9] Bugliarello, G. and Sevilla, J., 1970, velocity distribution and other characteristics of steady and pulsatile blood flow in fine class tubes, Biorheology 7, 85-107.

[10] Young, D.F., 1968, Effect of time dependent stenosis of flow through a tube. Trans. ASME. J. Eng. Ind., Vol. 90,248-254.

[11] Tondon, P.N. and Katiyar, V.K., 1976, Flow through a tube with time dependent stenosis, Medical and Life Science Engineering, Vol. 2, No. 3, 39-47.

[12] Forrester, J.H. and Young, D.F., 1970, Flow through a converging-diverging tube and its implications in occlusive vascular disease-I, 11, J. Biomech., 3, 297-316.

[13] Cavalcanti, S., 1995, Hemodynamics of an artery with mild stenosis, j. Biomechanics, 28, 387.

[14] Liu, H. and Yamaguchi, T., 2001, Waveform dependence of pulsatile flow in a stenosed channel, ASME. J. Biomech. Engg., 123, 88.

[15] Manica, R. And Bortoli, A.L., 2003, "Simulation of incompressible non-newtonian flows through channels with Suddens expansion using the power-law model", Tema-Tendencias em Matematica Aplicada e Computational, 4,pp. 333-340.

[16] Shanker, D.S. and Hemalatha, K., 2007, "A non-Newtonain fluid flow model for blood flow through a catheterized"- steady flow, Applied Mathematical Modeling, 31, pp. 1847-1864.

[17] Massoudi, M. And Phoue, T.X., 2008, "Pulsatile flow of blood using a modified second-grade fluid model", Computers and Mathematics with application, 56,pp. 199-211.

[18] Ismail, Z., Abdullah, I., Mustapha, N. And Amian, N., 2008, "A power-law model of blood flow through a tapered overlapping stenosed artery", Applied Mathematics and Computation, 195,pp. 669-680.

[19] Uday Shankar Chakraborty, 2011, Pulsatile blood flow through an inclined stenosed tube with a velocity slip, Advances and Applications in Fluid Mechanics, vol. 10,123-140.

[20] Riahi, Daniel N and Roy, Ranadhir, 2012, "Unsteady blood flow in an artery with an overlapping stenosis", International Journal of Applied Mechanics, 4, 02.

[21] Kumar, S. And Diwakar, C.S. 2012, A mathematical model for Newtonian blood flow in the presence of applied magnetic field, Aryabhatta Journal of Math & Informatics, Vol. 4, 265-278.

[22] R.Ali, Ramindar Kaur, V.K.Katiyar and M.P.Singh, 2010, Mathematical modelling for the flow of blood cells in capillaries, Advances in applied mathematical biosciences, vol.1, no 1,19-28.

[23] Mohan, V., Prasad, V., Varshney, N.K. and Gupta, P.K., 2013, Effect of Magnetic Field on Blood Flow (Elastico-Viscous) Under Periodic Body Acceleration in Porous Medium, IOSR Journal of Mathematics, 6, 43-48.