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Mathematical Model for Glucose-Insulin Regulatory System of Diabetes Mellitus

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Abstract

This paper works a new approach to regulate the blood glucose level of diabetes. We proposed a new mathematical model for the study of diabetes mellitus. The model takes into account all plasma glucose concentration, generalized insulin and plasma insulin concentration. The numerical solution presents the complex situation of diabetic patients. Computer simulations are used to evaluate the effectiveness of the proposed work.

Keywords: diabetes mellitus, glucose-insulin regulatory system, mathematical model.

Introduction

It is now commonly admitted that diabetes is sweeping the globe as a silent epidemic largely contributing to the growing burden of non-communicable diseases and mainly encouraged by decreasing levels of activity and increasing prevalence of obesity. The recent reports released by the World Health Organization [1] and the International Diabetes Federation [2] Diabetes mellitus is a disease in glucose-insulin endocrine metabolic system characterized by hyperglycemia resulting from defects in insulin secretion, insulin action or both. The two most common forms of diabetes are due to either a diminished production of insulin (Type 1 diabetes), or diminished response by the body to insulin (Type 2 diabetes). Both lead to hyperglycemia, which largely causes the acute signs of diabetes: excessive urine production, resulting compensatory thirst and increased fluid intake, blurred vision, unexplained weight loss, lethargy and changes in energy metabolism. In 1980 expert committee of WHO (World Health Organization) proposed 2 classification of Diabetes Mellitus and named it as IDDM (Insulin Dependent Diabetes Mellitus) or Type1 and NIDDM (Non Insulin Dependent Diabetes Mellitus) or Type2.But in 1985 Type 1 and Type 2 names were omitted and only IDDM & NIDDM were known as the types of Diabetes Mellitus [3]. Human bodies need to maintain glucose concentration level in a narrow range 70-110 mg/dl. If one's glucose concentration level is significantly out of the normal range, this person is considered to have the plasma glucose problem. With or without diabetes, monitoring of blood sugar levels is crucial to a person's health. Diabetes is a metabolic illness that can affect other organs of the body. Acquiring diabetes may aggravate a simple disease too. Internal organs like heart, kidney, lungs, liver, pancreas, or even the limbs are the favorite target of this metabolic disease. Senses may also be affected. Normally, it's the sense of sight that is targeted. Occurrence of diabetes must not create panic if monitored carefully.

Diabetes will remain as elevated blood sugar levels if treated properly. But if it starts to reach other organs or other processes in the body, then it's time to be more cautious. The basic thing that a patient or a normal person must know is blood sugar level. Having stored knowledge will make it easy for a person to react properly regarding diabetes at a given time.

Normally, blood glucose is at 64.8 to 104.4 mg/dl. The mean normal blood glucose level is 72 mg/dl. But it is important to note that blood glucose level is lowest in the morning before the first meal is taken. Two to three hours after a meal, blood glucose will elevate depending on what kind of food was taken. For a normal person without diabetes, blood sugar levels reach as high as 175 mg/dl after a meal is taken. This level will return to normal rate after some time. The time consumed for the blood sugar level to return to normal rate is faster in normal healthy person than in person with diabetes.

Diabetics must maintain a blood sugar level of 180 mg/dl after eating. In diabetics, the measurement is done two hours after meal. Higher levels than 180 mg/dl indicates that the person has taken too much carbohydrates or fatty food. Classic symptoms of diabetes mellitus such as frequent urination, excessive thirst and fatigue accompany these laboratory results in making the diagnosis.

Diabetes is associated with a large number of abnormalities in insulin metabolism, ranging from an absolute deficiency to a combination of deficiency and resistance, causing an inability to dispose glucose from the blood stream. Three factors: Insulin sensitivity, Glucose effectiveness, and pancreatic responsiveness, referred to in Pacini and Bergman [4], play an important role for glucose disposal. Failure in any of these may lead to impaired glucose tolerance, or, if severe, diabetes. The literature dealing with modeling for diabetes is mainly concerned with glucose and insulin dynamics.

Mathematical Model

Mathematical models have provided one mean of understanding Diabetes dynamics. There are various models based on glucose and insulin distributions and those models have been used to explain glucose/insulin interaction. All these models are valid under certain conditions and assumption. Although these models may be useful in research setting, they all have limitations in predicting blood glucose in real-time clinical situation because of the inherent requirement of frequently updated information about the models variable like glucose loads and insulin availability [6]. Consider a mathematical model comprised of glucose level G, glucose uptake activity

X and insulin level I. Many parameters have been taken and on the basis of these parameters values a mathematical model is formed. This model includes the basal values also i.e. G_b and I_b. The modal is defined as:

$$\frac{dG}{dt} = -m_1 G + m_2 I + m_1 G_b$$

$$\frac{dX}{dt} = -m_2 X + m_3 I - m_3 I_b + m_6 I_b$$

$$\frac{dI}{dt} = -m_3 I + m_4 G + m_4 m_5 - m_6 I + m_6 I_b$$

All the variables and parameters values used in mathematical models are described as:

G (t) The plasma glucose concentration at time *t (mg/dl)*
X (t) The generalized insulin variable for the remote compartment *(min-1)*
I (t) The plasma insulin concentration at time *t* *(µU/ml)*
Gb This is the basal preinjection value of plasma glucose *(mg/dl)*
Ib This is the basal preinjection value of plasma insulin *(µU/ml)*
m_1 Insulin independent rate constant of glucose rate uptake in muscles, liver and adipose tissue ($\min^{-1}$).
m_2 The rate of decrease in tissue glucose uptake ability ($\min^{-1}$).
m_3 The insulin independent increase in glucose uptake ability in tissue per unit of insulin concentration *Ib* ($\min^{-2}$ *(µU/ml)*).
m_4 The rate of the pancreatic β-cells' release of insulin after the glucose injection and with glucose concentration above *h [(µU/ml) min-2 (mg/dl)-1]*
m_5 The threshold value of glucose above which the pancreatic β-cells release insulin.
m_6 The first order decay rate for insulin in plasma (min-1) pancreatic β-cells release insulin

Numerical Solution

The analysis is done on the normal person as well as on different types of diabetic patient i.e patient 1, patient 2, patient 3.Basically there are patients who are suffering from Diabetes mellitus but the results from each patient is different and it is explained with the help of graphs and parameters values. Glucose is given to the patients then we studied the plasma glucose concentration, plasma insulin concentration and generalized insulin variable in the body of patients. The graph for these types of patients is obtained.

There are some variables and symbols used in the graphs:

G(t) The plasma glucose concentration at time *t* (mg/dl) "*"

X(t) The generalized insulin variable for the remote compartment *(min-1) “.”*
I(t) The plasma insulin concentration at time *t (μU/ml) “o”*

Data for Normal Person

The study of Glucose, insulin, and plasma concentration is done on the normal persons. The study of this case is shown by the graph(fig.1). Fig.1 shows the study for 10 hours. It show that when glucose is given to the normal persons the glucose concentration level become very high and as time passes the level become stable.The same can be seen in case of plasma insulin concentration but when we see the graph for generalized insulin variable there is no change even after some time it will remain same. The values of parameters for normal person are given in Table: 1.

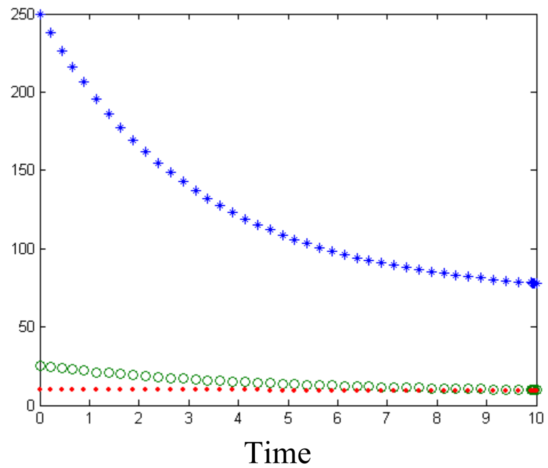


Figure 1: Glucose-insulin regulatory system (a)(*) Plasma glucose Concentration (b) Plasma insulin Concentration (o) (c) generalized insulin variable (.).

Table 1

Parameter	Values
m_1	0.0317000
m_2	0.0123
m_3	4.92×10^{-6}
m_4	0.0039
m_5	79.0353
m_6	0.2659
G_b	80
I_b	7

Data gor Patient 1

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patients. The study of this case is shown by the graph (fig.2). Fig.2 shows the study for 10 hours. It show that show that initially the glucose level is very high but after giving the glucose to the patient there is still no major fall in glucose level. After 10 hours from 250 mg/dl it falls to only about 275 mg/dl. But when we see the graph for generalized insulin variable as well as for plasma insulin concentration even after some time it is same and no change in its concentration level. The values of parameters for patient 1 is given in Table: 2

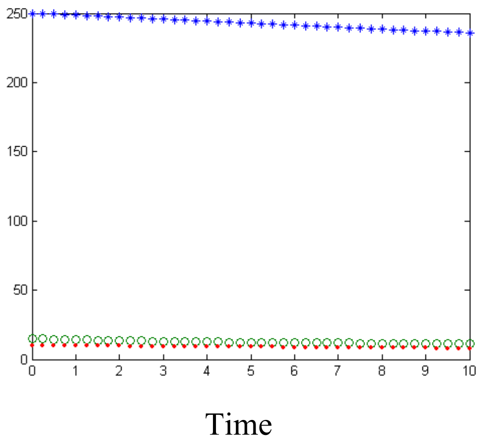


Figure 2: Glucose-insulin regulatory system (a)(*) Plasma glucose Concentration (b) Plasma insulin Concentration (o) (c) generalized insulin variable (.)

Table 2

Parameter	Values
m_1	0
m_2	0.017
m_3	5.3×10^{-6}
m_4	0.0042
m_5	80.25
m_6	0.264
Gb	80
Ib	7

Data for Patient 2

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patients of another type. The study of this case is shown by the graph (fig.3). Fig.3

shows the study for 10 hours. It show that show that initially the glucose level is very high but after giving the glucose to the patient 2 there is still no major fall in glucose level. In this graph plasma insulin concentration remains the same but when we see the graph for generalized insulin variable there is the minor fall after some time. The values of parameters for patient 2 are given in Table : 3.

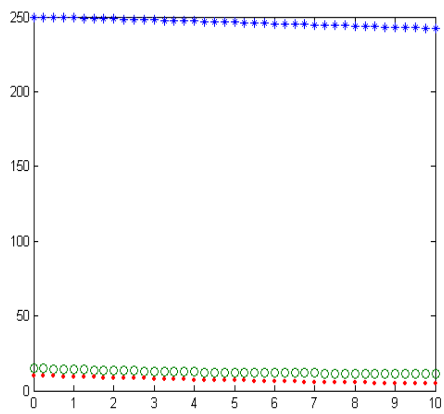


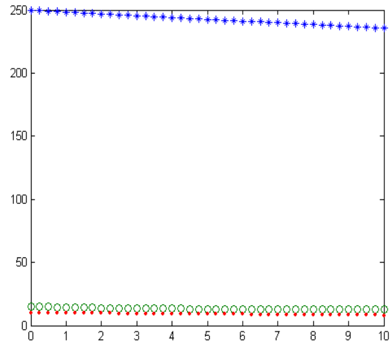
Figure 3: Glucose-insulin regulatory system (a)(*) Plasma glucose Concentration (b) Plasma insulin concentration(o) (c) generalized insulin variable(.).

Table 3

Parameter	Values
m_1	0
m_2	0.072
m_3	216×10^{-6}
m_4	0.0038
m_5	77.5783
m_6	0.2465
Gb	80
Ib	7

Data for Patient 3

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patient's type 3. The study of this case is shown by the graph (fig.4). Fig.4 shows the study for 10 hours. Again the level of glucose is same as in the case of other patients. In this graph plasma insulin concentration & generalized insulin variable have a minor fall after some time. The values of parameters for patient 3 are given in Table: 4.



Time

Figure 4: Glucose-insulin regulatory system (a)(*) Plasma glucose Concentration (b) Plasma insulin Concentration (o) (c) generalized insulin variable (.).

Table 4

Parameter	Values
m_1	0
m_2	0.0142
m_3	9.94×10^{-6}
m_4	0.0046
m_5	82.9370
m_6	0.2814
Gb	80
Ib	7

Discussion

In this paper a mathematical model is developed for different kind of diabetic patients. Different graphs are obtained for normal as well as diabetic a patient who shows the variability of Glucose, insulin and plasma glucose concentration. The model showed clearly the results given according to different scenarios. In conclusion, the present work presents and justifies a mathematical model of long-term diabetes progression. Diabetes management is one of important issues in the field of human regulatory systems. It shows the difference of glucose-insulin regulatory system, between a normal person and diabetic person. The glucose concentration of diabetic patient does not come down after a certain time which shows the evidence that the person suffer from diabetes. The current effort refines the previous model by Yasini et. al. (2009) and makes the resulting model directly useful for clinical purposes through a careful assessment of the relevant parameters.

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